Growth strategies: Fiscal versus institutional policies

Ingrid Ott\textsuperscript{a,b}, Susanne Soretz\textsuperscript{c,*}

\textsuperscript{a} Leuphana University of Lüneburg, Institute of Economics, Germany  
\textsuperscript{b} Hamburgisches WeltWirtschaftsInstitut (HWWI), Germany  
\textsuperscript{c} Ernst-Moritz-Arndt-University of Greifswald, Institute of Economics, Germany

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Abstract

This paper analyzes the growth impact of fiscal and institutional policies for alternative sizes of regions. The local government provides a public input that may be subject to relative congestion thus reducing its individual availability. Then private capital productivity is affected by the number of firms utilizing the governmental input. Institutional policies include the decision about the type of public input while fiscal policies decide on its extent. Private capital accumulation incurs adjustment costs that depend upon the ratio between private and public investment. After deriving the decentralized equilibrium, fiscal and institutional policies as well as their interdependencies and welfare implications are discussed. Due to the feedback effects both policies may not be determined independently. It is shown that depending on the region’s size a certain type of the public input maximizes growth. © 2007 Elsevier B.V. All rights reserved.

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1. Introduction

The fact that economic activity is not equally spread across space is well known and provides the basis for various fields of economic research to analyze corresponding causes and consequences: while growth theory focuses on debates about convergence/divergence as consequence of different growth rates, the new economic geography analyzes how the interaction between individuals in different regions shapes the economic landscape. In case of incomplete competition and increasing returns to scale of the technologies, economic activities tend to cluster, mainly in metropolitan regions, and these agglomerations in turn tend to gain importance as principle engines of growth. Consequently growth rates in these regions differ from those in the periphery and ground is set for further divergence (see Barro and Sala-I-Martín...
(2004), Krugman and Venables (1995) or Baldwin et al. (2003) for recent overviews over growth theories and economic geography). However, the recent report of the OECD (2006) on the competitiveness of cities in the global economy also shows that size is not automatically synonymous with economic success as measured by income per capita. Again, one of the main questions in economic analysis, namely should the government intervene the market process and how, remains relevant also at a regional level.

One of the main topics addressed during the last several years within growth economics has been to point at the fundamental role of institutions in the growth process. Institutions are referred to as economic, legal or social arrangements (see e.g. North (1990), Knack and Keefer (1995) or Acemoglu et al. (2001, 2005)). More precise formulations include the focus on property rights protection or regulations in financial, labor or product markets (see e.g. Eicher and Garcia-Penalosa (2006)). Most recent discussions refer to ‘appropriate’ institutions that are based on the seminal work of Gerschenkron (1962) and argue that different types of institutions or policies maximize growth (see e.g. Aghion and Howitt (2005), Rodrik (2005) or Acemoglu et al. (2006)).

This paper confers the discussion of the institutions’ impact on growth at a regional level and extends it with respect to its contribution within a comprehensive governmental policy mix. As growth determinant serves a governmentally provided public production input that – depending on its characteristics – may be interpreted e.g. as basic research, road networks or regional public good such as a harbor or an airport. The analysis is carried out in the context of an endogenous growth model that is applied to a regional level. The regions may be distinguished with respect to their size and possibly the type of productive input provided by the government. With this respect the model is based on the seminal work of Barro (1990) but explicitly focuses on the impact of the region’s size as measured by the number of firms utilizing the public input. In analogy to Turnovsky (1996) the public input acts as complement to private capital within the production process, increases private capital productivity and affects the firm’s adjustment costs. Institutional policies include the decision on the type of the governmental input, as characterized by the prevailing degree of congestion, as well as the choice of the economy’s size. The latter argument gains importance e.g. in the discussion about the EU enlargement or decisions at a regional level about the extent of metropolitan areas. The governmental input is provided free of charge to the individuals, hence taxes are levied to finance the provision.

The emphasis is laid on the relationship between specific governmental policies and regional growth. Since the public input may be congested, the region’s size is of major importance for the resulting policy options. With this it is possible to answer e.g. questions like: How big should an economy be given that a certain public input exists? Which public input should be provided by the local government for a given size of the region? Which growth impact is incorporated in certain fiscal or institutional policies respectively? Which determinants characterize an optimal policy mix? It is shown that politicians have substantial room for creatively packaging alternative instruments into a policy mix that is sensitive to local opportunities and constraints. In addition it becomes obvious that it is not possible to derive a unique policy recommendation but that regional peculiarities have to be carefully considered if the goal of maximizing growth is to be sustained.

The paper is organized as follows: After presenting the analytical framework in Section 2, the equilibrium in the decentralized economy is derived in Section 3 and its economic implications are discussed. Section 4 focuses on welfare and the interdependencies between fiscal and institutional policies. Subsequently numerical simulations are carried out since it is not possible to derive closed-form solutions of the model. Section 6 concludes while some formal details are relegated to the appendix.

2. The analytical framework

2.1. Regional aspects and production technology

We consider a multi-region economy in which a benchmark region with total population size normalized to \( N = 1 \) is used as numeraire. It is assumed that the other regions are populated with \( N > 1 \) identical individuals. This specification has two major advantages: It allows to (i) analyze the growth impact of governmental policies for regions of different sizes and (ii) to consider the scale impact of alternative institutional settings within the model. The latter argument will be detailed during the further presentation of the model. We focus on regional size as explicitly defined by the population size, \( N \), and its interdependence with regional growth. This allows to address issues like economic growth of mega-cities or congestion in populous regions.
Production is done by households who are consumers and producers of the homogenous output at the same time.\textsuperscript{1} Output of a such a single firm is determined by privately owned capital, \( k \), and the individually available amount of public services, \( G_e \).\textsuperscript{2} The individual’s production function
\[ y = f(k, G_e) \] (1)
is homogenous of degree one in the two inputs.

The governmental input is provided free of charge.\textsuperscript{3} According to the formulation in Barro (1990) we assume that individuals consider the total amount of government expenditures, \( G \), to be independent from their individual decisions. To finance the governmental input, the government can levy an income tax at rate \( \tau_c \) or a consumption tax at rate \( \tau_c \). Since we refrain from the decision about labor supply, the consumption tax amounts to a lump-sum tax which is growth neutral. In the subsequent analysis, we assume a balanced government budget: Whenever we consider a change in the governmental input, we assume that it is financed via a simultaneous adjustment of the consumption tax rate. Nevertheless, since government debt is growth neutral in the underlying endogenous growth model\textsuperscript{4} one could equivalently consider the case of government debt and assume that an increase in the governmental input is financed by the issue of government bonds.

The productive services derived by the representative individual from a given amount of public expenditure, \( G \), depend upon the usage of his individual capital stock relative to aggregate usage. This describes the situation of relative congestion that is introduced via a typical congestion function
\[ G_e = Gk^\sigma K^{-\sigma} \quad 0 \leq \sigma \] (2)
where \( K \) denotes the aggregate stock of private capital (see e. g. Barro and Sala-I-Martin (1992) or Eicher and Turnovsky (2000)). The case \( \sigma = 0 \) corresponds to a nonrival pure public input whereas \( \sigma = 1 \) reflects a situation of proportional (relative) congestion. Accordingly, the cases \( 0 < \sigma < 1 \) correspond to situations of partial (relative) congestion, in the sense that given the individual stock of capital, government spending can increase at slower rate than does \( K \) and still provide a fixed level of services to the firm. The situation \( \sigma = 0 \) may be interpreted as a situation in which the government provides a pure public good, e. g. basic research. Its usage by one firm does not affect possible usages of the others. In contrast to this, \( \sigma \leq 1 \) corresponds e. g. to the provision of a road network. In extreme it is proportionally congested, and each of the \( N \) individuals within the region may use \( G/N \) parts for production. As Turnovsky (1996, p. 364) argues, the case \( \sigma > 1 \) describes a situation where congestion is so great that the public input must grow faster than the economy in order for the level of services provided to the individual firm to remain constant. This case is unlikely at the aggregate level, but may well be plausible for local public goods (see Edwards (1990)). A local public good could be a harbor that is provided by the regional government. Nevertheless it also may be used by individuals coming from outside the region. Note that usually within this type of growth model population size is normalized to unity. This specification suppresses scale effects within the resulting growth rate that empirically seem to be well implausible at the aggregate level. Nevertheless, scale effects are due to the phenomenon that a public input characterized by partial rivalry can be used by additional firms at relatively low costs. Hence, the normalization is done at the cost of several other effects and policy implications which then are also hidden. This gap will be addressed within this paper.

We assume that the government sets its aggregate expenditure in proportion to the aggregate capital stock so that \( g = G/K \) in order to allow for steady state growth. Introducing (2) into (1) the production function may be rewritten to be linear in capital\textsuperscript{5}
\[ y = \alpha\left(gk^{\sigma-1}K^{1-\sigma}\right) \cdot k \quad \alpha > 0 \quad \alpha'' < 0 \] (3)
thus allowing for sustained growth. The term \( \alpha \) represents a convex productivity function.

\textsuperscript{1} Equivalently, we could formulate households who supply capital and one unit of labor, and buy the consumption good, and firms which pay for the used production factors and supply the consumption good. The results would not be affected.

\textsuperscript{2} With this specification of the production function the paper draws back on the seminal work of Barro (1990) and Edwards (1990). Extensions are carried out by Glomm and Ravikumar (1994a,b), or Fisher and Turnovsky (1998). An overview of the most relevant arguments is given in Turnovsky (2000a).

\textsuperscript{3} A discussion of the growth implications of a partly excludable public input can be found in Ott and Turnovsky (2006).

\textsuperscript{4} This is shown for a comparably structured model e. g. by Turnovsky (2000b, chapter 12).

\textsuperscript{5} The relationship between the general production function (1) and the linear form (3) can be found in Appendix A.
2.2. Capital accumulation and installation costs

The process of capital accumulation involves installation costs according to

\[ t \cdot (1 + \phi) = t \cdot \left(1 + \phi \left(\frac{t}{G_s}\right)\right) \quad \phi' > 0 \quad \phi'' \geq 0 \quad \phi(0) = 0 \]  

(4)

where \( t \) denotes private investment.\(^6\) The functional form \( \phi \) has to be homogenous of degree zero in the two arguments if a steady-state equilibrium having ongoing growth is to be sustained. Above, \( \phi \) implies that installation costs depend on private investment, \( t \), as well as on the governmental input, \( G_s \). Usually, authors who focus on capital adjustment costs model them as ratio between the investment in each period or time increment respectively and the firm’s capital stock. An exception is the paper of Turnovsky (1996) who develops a one-sector endogenous growth model in which capital investment incurs adjustment costs that are related to governmental activity. Since the individually available governmental input, \( G_s \), is a function of total public expenditure, individual and aggregate private capital as well as of the degree of congestion, installation costs are affected via several channels: Due to \( \phi' > 0 \) an increase in private investment, \( t \), rises installation costs whereas they are reduced with an increase in \( G_s \). Consequently, all parameters of (2) also affect the installation costs. All things being equal, \( G_s \) increases with \( G \) and \( k \). This results directly from the complementarity of both inputs within (2). Hence, installation costs of a given amount of investment decrease with a rise in the individual capital stock, \( k \), or public investment, \( G \). This assumption resembles the usual finding that the higher the capital stock of a firm, the lower are the installation costs. Moreover, installation costs are lower in an opportune industrial area (higher \( G \)). The contrary is true for aggregate capital, \( K \), whereas the impact of congestion is more sophisticated; details are discussed below. Note that in contrast to private capital, the aggregate capital is exogenous to the individual firm. Private investment, \( t \), determines capital accumulation according to

\[ t = k + \delta k \]

(5)

with the rate of capital depreciation denoted by \( \delta \).

2.3. Institutional and fiscal policies

As usual within this type of growth model we refer to fiscal policies with respect to those activities that affect tax policies or the fixing of a certain amount of the governmental input. We extend the corresponding policy implications with respect to institutional arguments thereby following Turnovsky (1996, p. 363) who argues:

‘… the degree of congestion is to some extent the outcome of a policy decision, and once determined, the degree of congestion turns out to be a critical determinant of optimal tax policy.’

This statement makes clear that governmental policy by all means includes fiscal policy.\(^7\) But, over and above, governmental policy also affects the prevailing degree of congestion. This is done via the provision of a certain type of governmental input, e.g. basic research or a road network or a regional public good. To keep the analysis simple we assume that each region is only equipped with one type of governmental input.

Furthermore, we follow North (1990) who argues that institutions cover a variety of different economic, social and legal arrangements that partly result from the political process. To provide a framework for the discussion we combine these arguments and interpret the actual degree of congestion as being the outcome of alternative institutional settings. In addition to this they also include efforts of the government with respect to the size of the region as determined by the number of firms. In the following part of the paper we refer to institutional policies as to those that determine the type of public input that is provided by the government\(^8\) and to regulations concerning the extent of the region. To sum up the formal implementation of alternative governmental policies: Fiscal policies tie

\[ 6 \] With this specification our model relies the Tobin q investment theory that focuses on the impact of adjustment costs (see Hayashi (1982)). A survey of relevant approaches is given by Hamermesh and Pfann (1996) or Cooper and Haltiwanger (2006) whereas recent empirical studies can be found in Hall (2004). An industry specific discussion is done by Caballero and Engel (1999).

\[ 7 \] The corresponding growth impact is discussed in detail in the cited paper.

\[ 8 \] Anyhow, the degree of congestion is still exogenous and we do not endogenize institutional policies but analyze the impact of alternative (exogenously given) institutional settings on the region’s performance.
up to the provision of \( G \) and the underlying tax rates whereas institutional policies focus on the impact of alternative levels of \( \sigma \) and/or the region’s size, \( N \).

Fig. 1 summarizes the coherences within the model. The individually available public input, \( G_s \), as well as its constituents, \( G, k, K \) and \( \sigma \), has a significant impact on both output and installation costs (see (1) and (4)). This is illustrated by the vertical arrows. Within each function the signs below the constituents expose either a positive or a negative impact on the dependent variable. Regional and institutional arguments enter the model as illustrated by the dashed box. Institutional arrangements are represented by the degree of rivalry, \( \sigma \), and comparisons of alternative regions may be executed by considerations of alternative sizes of \( N \).

2.4. Lifetime utility, individual resource constraint, and private investment

The infinitely lived representative individual maximizes the intertemporal utility function

\[
U = \int_0^\infty e^{-\rho t} \frac{e^{1-1/\varepsilon}}{1-1/\varepsilon} dt
\]

with constant utility discounting, \( \rho > 0 \), and constant intertemporal elasticity of substitution, \( \varepsilon \).\(^9\) The individual decides about the utility maximizing consumption path according to the budget constraint

\[
(1 - \tau_y)y = (1 + \tau_c)c + (1 - \tau_i)i \cdot (1 + \phi)
\]

where \( \tau_y \) is the income tax rate, \( \tau_c \) denotes the consumption tax rate, and \( \tau_i \) is the investment subsidy rate. The fiscal parameters are set by the government and are considered to be exogenous and constant within individual utility maximization.

3. Decentralized economy

The individual’s problem is to choose the rate of consumption, \( c \), of investment, \( i \), and of capital accumulation, \( k \), to maximize (6) subject to the budget constraint (7) and capital accumulation (5). The intertemporal maximization problem results in the Hamiltonian

\[
H = e^{-\rho t} \left[ \frac{c^{1-1/\varepsilon}}{1-1/\varepsilon} + \lambda \left[ (1 - \tau_y)f - (1 + \tau_c)c - (1 - \tau_i)i(1 + \phi) \right] + q \lambda \left( 1 - \delta k - k \right) \right]
\]

\(^9\) This specification of individual utility, which is quite usual in growth theory, is necessary in order to allow for steady state growth.
where $\lambda$ is the shadow value of wealth in the form of new output and $q\lambda$ is the shadow value of the agent’s capital stock. Analysis of the model is simplified by using the shadow value of wealth as numeraire. Consequently, $q$ is defined to be the market value of capital in terms of the (unitary) price of new output.

The necessary conditions which determine optimal consumption, investment and capital accumulation result in

$$e^{-1/\varepsilon} = \lambda(1 + \tau_c)$$  \hspace{1cm} (9a)

$$(1 - \tau_c)(1 + \phi + t\phi) = q$$  \hspace{1cm} (9b)

$$\frac{(1 - \tau_c)f_k}{q} - \frac{(1 - \tau_c)t\phi_k}{q} + \frac{\dot{q}}{q} - \delta = \rho - \frac{\dot{\lambda}}{\lambda}$$  \hspace{1cm} (9c)

To fully specify the first order conditions, the transversality condition

$$\lim_{t \to \infty} q\lambda e^{-\alpha t} = 0$$  \hspace{1cm} (10)

has to be met, too.

Condition (9a) equates marginal utility of consumption to the shadow value of wealth, $\lambda$, which is given in units of new output. Condition (9b) equates the marginal installation costs to the market value of capital, $q$. Since marginal investment costs are nondecreasing with $t\phi$, either an increase in private investment or a decrease in the available public input raises the equilibrium market value of capital, $\partial q/\partial(t\phi) > 0$. Condition (9c) determines optimal capital accumulation and reflects the results of the standard Keynes–Ramsey-Rule. Marginal return on consumption (RHS) is equilibrated with the rate of return on acquiring an additional unit of physical capital (LHS). The return on an additional unit of physical capital is composed of the following elements: (i) $(1 - \tau_c)f_k/q$: after tax output per unit of installed capital (valued at the price $q$), (ii) $\dot{q}/q - \delta$: (net) rate of capital gain, (iii) $(1 - \tau_c)t\phi_k/q$: reflects the fact that an additional resource of benefits of a higher capital stock is to reduce the installation costs associated with future investment.

We now derive the equilibrium growth rate of the decentralized economy. The representative agent in making his individual investment decision assumes that he has a negligible impact on the aggregate capital stock and therefore ignores the linkage between its own investment decision and the resulting aggregate capital stock. This misperception is the source of the congestion externality generated by capital accumulation. Formally this may be illustrated by the way how firms perceive their individually available amount of the public input during the process of capital accumulation. Starting from (2) it follows

$$\frac{\partial G_s}{\partial k} = \sigma \frac{G_s}{k} \geq 0 \quad \text{if} \quad \sigma \geq 0.$$  \hspace{1cm} (11)

The firms’ usage seems to increase with a rise in the individual physical capital. This positive effect is influenced via three channels: (i) the absolute size of the government, $G$, (ii) the prevailing degree of congestion, $\sigma$, and (iii) the scale of the economy, $N$. While the first effect is widely discussed within the congestion literature, the analysis usually is reduced to situations where $\sigma \leq 1$ and $N = 1$. Most of the existing models refrain from making statements about the economic impact of local public goods and of the size of the economy.
The economy’s growth rate may be derived from the first order conditions (9a)–(9c) and \( \hat{\lambda} \) from (9a) which provides \( \hat{c} \). A closed form solution of the model requires a specification of installation costs. Following Barro and Sala-I-Martin (2004, chap. 5) we assume

\[
\frac{\dot{c}}{G_s} = b \frac{1}{G_s}, \quad b \geq 0.
\]  

(12)

Hence installation costs are quadratic in \( t \) and amount to

\[
\dot{t} \cdot (1 + \phi) = t \cdot \left( 1 + b \frac{1}{G_s} \right).
\]  

(13)

Additionally, we use (9b) to receive the relationship between \( \frac{G}{G_s} \) and \( q \)

\[
\frac{t}{G_s} = \frac{q/(1 - \tau_i) - 1}{2b}.
\]  

(14)

With this the decentralized growth rate results as

\[
\hat{c} = \hat{\lambda} \left( \frac{(1 - \tau) f_k}{q} + \frac{(q - (1 - \tau_i)) g \sigma N^{1-s}}{(1 - \tau_i) q 4b} - \delta - \rho + \frac{\dot{q}}{q} \right).
\]  

(15)

It is composed of the net marginal product of capital including adjustment costs, the rate of time preference and the growth rate of the market value of capital. The latter is proved to be constant in equilibrium as a situation in which all macroeconomic variables grow at a constant and equal rate.

To specify the market value of capital, \( q \), Eq. (9c), \( \hat{\lambda} \) from (9a) and (14) are used. This provides the equation of motion of \( q \) as

\[
\dot{q} = \frac{(2 - \varepsilon \sigma) g N^{1-s}}{4 b (1 - \tau_i)} q^2 + \left( \rho - \frac{1 - \varepsilon}{\varepsilon} \delta - \frac{(1 - \varepsilon \sigma) g N^{1-s}}{2 \varepsilon b} \right) q - \left( (1 - \tau_i) f_k + \frac{(1 - \tau_i) \sigma g N^{1-s}}{4b} \right).
\]  

(16)

In equilibrium the market value of capital is constant. Hence, its derivative with respect to time, \( \dot{q} \), must be zero. Equalizing (16) to zero and solving the resulting quadratic equation for \( q \) implies

\[
q^d = \frac{1 - \tau_i}{2 - \varepsilon \sigma} \left[ 1 - \varepsilon \sigma - \frac{2b (\varepsilon \rho - (1 - \varepsilon) \delta)}{g N^{1-s}} + \sqrt{\left( 1 - \varepsilon \sigma - \frac{2b (\varepsilon \rho - (1 - \varepsilon) \delta)}{g N^{1-s}} \right)^2 + \frac{e(2 - \varepsilon \sigma)}{1 - \tau_i} \left( \frac{4b (1 - \tau_i) f_k}{g N^{1-s}} + (1 - \tau_i) \sigma \right)} \right].
\]  

(17)

Altogether, the decentralized equilibrium is then defined by the growth rate (15) together with \( q^d \) from (17). Due to the discussed externality, decentralized capital accumulation it is not optimal but fiscal and/or institutional policies would be required in order to internalize the market failures.

4. Interdependencies between fiscal and institutional policies and welfare

To assess the welfare implications of the decentralized equilibrium it is necessary to compare it to the first–best optimum. Causal for the deviation between first–best and market equilibrium are different perceptions of the congestion functions by the individuals and the benevolent social planner (see (2) and (C.1)). To keep the presentation concise, Table 1 provides the most important equations. Since the underlying mathematics are analogous to those in the

\[\text{footnote: The quadratic Eq. (17) has two solutions. Within Appendix B it is shown that only } q^d \text{ as presented by (16) is a feasible solution.}\]
last section, we relegate them into Appendix C and focus on a comparison of the results. As illustrated in Section 2 the discussion incorporates the impact of fiscal and institutional policies (changes in \( g, N, \sigma \)) on the production effect whenever capital productivity is concerned whereas the adjustment cost effect refers to implications of the analyzed policies for the adjustment costs.

The optimal government expenditure rate, \( \dot{g}^* \), may not be determined explicitly but it is nevertheless worthwhile to think about it for the following reasons: It is possible (i) to compare the government’s expenditure rate to the one in an economy without adjustment costs and (ii) to illustrate the interaction between fiscal and institutional policies. Formally an optimal government size requires

\[
\alpha' \left( g^* N^{1-\sigma} \right) + \frac{(q-1)^2}{4b} N^{1-\sigma} = 1.
\]

The left-hand side measures the welfare benefits of a unit increase in government expenditure. These include: (i) the marginal benefits to the productivity of existing capital, \( f_g/k \) (see (C.6)), and (ii) the marginal benefits from reducing the costs associated with installing new capital. An optimum requires that these marginal benefits equal the unit resource costs they absorb. Note that if \( N = 1 \), the optimal government size is independent of the degree of rivalry.\(^{14} \) If instead \( N > 1 \), both the population size and the nature of the public input as incorporated within the term \( N^{1-\sigma} \) determine the resulting value of \( g^* \).

Consequently it is possible to analyze the impact of institutional policies – as formalized by changes of \( N \) and/or \( \sigma \) – on the optimal fiscal policy – as formalized by \( g \). Or put differently, since \( g^* \) is affected by the size of the economy and the type of governmental input, it is not possible to determine fiscal policy independent of institutional decisions. Taking a more precise look at the linkages within the model two cases are to be distinguished:

(i) \( \phi = 0 \): In case that the governmental input has no impact on investment costs, the corresponding benefits do not arise, and optimal governmental expenditure is given if \( \alpha' \left( g^* N^{1-\sigma} \right) = 1 \). Increases (decreases) in \( N^{1-\sigma} \) then induce a decreased (increased) equilibrium level \( g^* \). Alternative types of governmental inputs, as represented by different degrees of congestion, affect the optimal governmental size: Given \( N > 1 \), an increase in congestion leads to an increase in the optimal governmental size. The higher the degree of rivalry of the provided public input, the larger is the optimal amount of the public input in order to compensate for decreased individual availability. Aside from this, the nature of the public input is important since \( N^{1-\sigma} \) increases (decreases) with \( N \) as long as \( \sigma < 1 \). As long as the public input is partially congested (\( \sigma < 1 \)), the total amount of the public input can grow at a smaller rate than population does (and hence aggregate capital) in order to keep the individually available amount

\(^{14} \) This is a well-known result within congestion models that normalize population size to unity.
of the public input constant. Hence the optimal relative governmental size \( g^* \) decreases with a rise in \( N \) if the public expenditure is partially congested whereas \( g^* \) rises with \( N \) in case of regional public inputs.

(ii) \( \phi > 0 \): If adjustment costs arise, the marginal benefits from reduced installation costs allow for a decrease of the required marginal benefits from capital. The term \( \alpha' \) is reduced thus increasing \( g^* \). Hence compared to a situation without adjustment costs, the optimal government size is increased since the public input does not only act as production input but also reduces adjustment costs. With respect to the impact of the nature of the public input the argumentation from above continues to hold. To sum up: adjustment costs bring about increases of the optimal size of the governmental input. As long as we allow for alternative regional sizes \( (N > 1) \) also institutional policies affect the optimal size of the governmental input and with this fiscal policy.\(^{15}\)

The first–best optimum is composed of the optimal size of the government, \( g^* \), and the first–best growth rate\(^{16}\)

\[
(c)^* = e \left[ \frac{\alpha (g^* N^{1-\sigma}) (1 - \eta^*)}{q} - \delta - \rho \right]
\]

Both, \( g^* \) and \( c^* \), are affected by the market value of capital, \( q^* \), that also has to be realized. It is given by

\[
q^* = \frac{\alpha (g^* N^{1-\sigma}) \left( \epsilon \left( 1 - \eta_{ys}^* \right) + 2\eta^* \right) - 2g^*(1 - N^{1-\sigma})}{\epsilon \rho - (1 - \epsilon) \delta + 2g^* N^{1-\sigma}}
\]

**Welfare implications**

Henceforth we assume that the expenditure ratio is set optimally, \( g = g^* \), neglect tax rates and focus on the impact of alternative degrees of congestion and/or sizes of the region. Or put differently: We analyze the implications of alternative institutional policies within the model. In doing so we begin with the analysis of the components of both growth rates, \( \dot{c} \) and \( \dot{c}^* \), as given by (15) and (19). These include identical rates of time preference, \( \rho \), and depreciation, \( \delta \). The arising differences are due to alternative perceptions of the congestion function and affect the marginal product of capital, \( f_k \) (thus inducing a production effect), as well as the market value of capital, \( q \), and the marginal reduction of future adjustment costs, \( \phi_{bk} \), in various ways (thus inducing an adjustment cost effect).

(i) The marginal product of capital, as perceived by the individuals, exceeds the optimal one whenever the public input is characterized by rivalry. The gap between both unequivocally increases with \( \sigma \) since this spurs the incentive to accumulate capital: the individually available amount of the public input, \( G_\alpha \), seems to be related stronger to the individual capital stock.\(^{17}\)

(ii) The decentralized market value of capital coincides with the optimal one if and only if the social capital return equals the private capital return and if the public production input is characterized by proportional congestion \( (\sigma = 1) \). This requires a very specific combination of fiscal and institutional policies. The usage of the entire public input is equally distributed among the individuals and each may use \( 1/N \) parts. If \( \sigma \neq 1 \) and all things being equal the prevailing degree of congestion drives a wedge between \( q^d \) and \( q^* \).

(iii) The static dimension of the adjustment costs as incorporated by \( \phi_\alpha \) is directly affected by the actual level of the market value of capital, \( q \). The optimal and the decentralized values only coincide under the very specific assumption \( q^* = q^d \), and the above statements again become relevant.

\(^{15}\) Note that since the optimal value of \( g \) may not be determined explicitly it is not possible to find a closed–form solution of the first–best optimum. We work on this problem in Section 5 where we specify the production technology by a Cobb–Douglas production function.

\(^{16}\) The term \( \eta_{ys}^* = \dot{c}_y/\dot{c}_g y/y = \dot{c}_\alpha/\dot{c}_g \ y/\alpha \) within the growth rate denotes the partial production elasticity of the public input.

\(^{17}\) As argued above, this is the consequence of the external effect of capital accumulation as discussed within the congestion literature. If congestion is proportional and given that labor is supplied inelastically, a distortionary income tax may be used to reduce the private capital accumulation activity. The corresponding governmental revenues suffice to finance the optimal amount of the public input (see e. g. Barro (1990) or Turnovsky (2000a)).
(iv) The dynamic dimension of adjustment costs (namely reduced costs in the future), as measured by $\phi_k$, may be suboptimally high or low, depending on the level of $q$ and the prevailing degree of congestion. All things being equal and given that $q^* = q_d$, the following results can be derived: As long as $\sigma < 1$ the corresponding ratio between private and public investment, $\iota/G_s$, is suboptimally low thus spurring growth. But note that this effect becomes weaker with increasing $\sigma$. If the public input is proportionally congested, $\sigma = 1$, the optimal ratio between private and public investment is realized and the individuals correctly perceive $\phi_k$. However, if the governmental input is a regional public good, $\sigma > 1$, the individuals perceive the ratio $\iota/G_s$ as being higher than the actual one; hence they do not realize future reductions of investment costs up to their full extent. This chokes capital accumulation and thus reduces the growth rate. Consequently the growth impact of future adjustment cost reductions is considerably determined by the nature of the public input.

To sum up

Aside from positive growth effects of congestion, as those that arise in the context of capital productivity, also negative effects of congestion may be identified. Neither for the general nor for the linear production technologies it is possible to derive the optimal value of $g^*$. Given this we are not able to explicitly formulate the first best optimum. However, the usual argumentation as carried out within the majority of growth models continues to hold: Congestion induces externalities thus driving a wedge between decentralized and first-best optimum. Fiscal policy in form of taxes may then be used in order to correct for the market distortions and additionally to finance the provision of $g^*$. Hence any optimal fiscal policy must also correct for any arising externalities and not only take care about the efficient provision of the optimal amount of the governmental input.

5. Numerical simulations

Regional policies frequently pursue the goal to maximize the growth rate. In the context of our model this could be achieved via several channels, namely fiscal and institutional policies. We now focus on the implications of alternative governmental sizes, $g$, and institutional arrangements, $\sigma$ and/or $N$, and on the decentralized equilibrium as given by (15) and (17). The numerical simulations will be carried out for the Cobb–Douglas production technology

$$y = A k^{\beta} G_s^{1-\beta} \quad 0<\beta<1 \quad A>0$$

where $A$ denotes the technological level of the economy. The decentralized marginal product of capital then amounts to

$$f_k = [\beta + \sigma(1-\beta)]A \cdot (gN^{1-\sigma})^{1-\beta}.$$  

To illustrate the different effects of governmental policy on the equilibrium growth rate we use the parameter specifications in Table 2.

---

18 This is hardly amazing as it follows the usual logic of the congestion models.
5.1. Economic impact of governmental size

We begin the discussion with an analysis of the impact of alternative sizes of the government, $g$, on the resulting market value of capital, $q^d$, and on the growth rate, $\hat{c}$. Due to the relation $g = G/K = G/Y \times Y/K$, the level of $g$ may be interpreted as representing the expenditure ratio. We assume a capital coefficient equal to $K/Y = 4$. Then an expenditure ratio $G/Y = 0.25$ corresponds to $g = 0.0625$ whereas $G/Y = 0.5$ is represented by $g = 0.125$. A graphical illustration for alternative sizes of the economy is given within Fig. 2. Bold lines represent a public input as local public good ($\sigma = 1.5$) whereas thin lines reflect pure public goods ($\sigma = 0$).

![Graphs](image)

Fig. 2. The impact of governmental size, $g$, on the level of $q$ and $\hat{c}$. Bold lines correspond to local public goods ($\sigma = 1.5$) whereas thin lines reflect pure public goods ($\sigma = 0$).

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\[\text{It is also possible to analyze even higher or lower levels of the expenditure ratio. The structural results of the model would not be affected.}\]
As can be seen within Fig. 2(a), (c) and (e), the equilibrium market value of capital decreases with a rising expenditure ratio. This relationship holds for all sizes of the economy and for all types of public goods. Bigger governments imply a higher level of $G_s$ and reduces the ratio between private and public investment, $v/G_s$. This lowers the adjustment costs, $\phi$, and the market value of capital, $q^d$, is reduced. Independent of the size of the economy the equilibrium value of capital is always higher in case of the uncongested public input than in case of a regional public good (bold lines always above thin lines). This is caused by the fact that increased rivalry reduces the ratio $v/G_s$, thus increases $\phi$ and with this $q^d$. Furthermore, the wedge between both functions in each subfigure increases with the size of the economy. This result is due to value of $\sigma = 1$ that separates the less than proportionally congested inputs from regional public goods: If $\sigma < 1$, an increase in $N$ also increases $G$ since the ratio $g = G/K$ is constant. Then each firm disposes of more governmental input than before as long as congestion is less than proportional. This reduces the decentralized market value of capital, $q^d$. The thin lines shift downwards with an increased size of the economy. The opposite applies in case of the governmental input as regional public good, $\sigma > 1$. Then an increase in $N$ reduces the individually available amount of the public input thus ending up in a higher market value of capital, $q^d$. The bold lines shift upwards and altogether the wedge between both lines rises.

The corresponding relationships between the growth rate and the government’s size can be seen in Fig. 2(b), (d) and (f). Again, $\dot{c}$ unequivocally increases with $g$. This is due to the complementarity of both production inputs: $f_k$ unequivocally rises with $g$. But depending on the economy’s size, either the growth impact of a regional public good ($N = 1$: bold line above thin line) or of the pure public good ($N = 3$: thin line above bold line) overweighs. This result is due to the ambiguous productivity effect: Taking the first derivative of (22) with respect to the degree of rivalry illustrates $\partial c_k/\partial \sigma \geq 0 \iff \ln(N) \geq 1/(\beta + \sigma(1 - \beta))$. There exists a ‘critical size’ of the economy, determined by $\ln N$ and in the following denoted by $\tilde{N}$ that determines whether increased rivalry enhances or reduces private capital productivity. Similarly it is possible to derive a ‘critical degree of congestion’, in the following denoted by $\tilde{\sigma}$ Capital productivity is an essential part of the growth rate and it is possible to illustrate the ambiguity of the growth effect also by simulation. For the assumed calibration parameters the critical values are given by $\tilde{N} (\sigma = 0) = 4.17$ and $\tilde{N} (\sigma = 1.5) = 2.39$ respectively. Analogously, it is possible to determine a critical size of the degree of congestion, denoted by $\tilde{\sigma} = 1/(1 - \beta) [1/(\ln N) - \beta]$. It is given by $\tilde{\sigma}(N = 3) = 0.70$ and $\tilde{\sigma}(N = 2) = 2.48$ respectively. If $N$ lies below the ‘critical value’, $\tilde{N}$, the productivity enhancing effect of an increased size of the economy dominates. In case of a sufficiently big region ($N > \tilde{N}$) the negative scale effect within $f_k$ becomes dominant. Since this effect is reinforced by $\sigma$, the growth effect of a regional public good is smaller than it is the case if $G_s$ is a pure public production input.

5.2. Economic impact of institutional arrangements

Within governmental policies, the determinants of institutional arrangements also have to be chosen and additionally affect the equilibrium. These arrangements may be interpreted as the prevailing degree of congestion and/or the size of the economy, i.e. the government may decide about the nature of the public input provided or on its endeavors to settle firms. In the context of this approach it is also possible to analyze the impact of institutional changes on the resulting equilibrium. Again we focus on the market value of capital as well as on the growth rate and discuss how they are determined via alternative $\sigma$ and $N$. A graphical illustration can be found in Fig. 3. Bold lines correspond to ‘big’ governments ($g = 0.125$) whereas thin lines reflect ‘lean’ governments ($g = 0.0625$) as introduced above.

As can be seen within Fig. 3(a), (c) and (e), the market value of capital in the decentralized economy increases with $\sigma$ for all sizes of the economy. Again the intuition behind this result is as follows: a rise in rivalry reduces the individual available amount of the public input, this increases the ratio between private and public investment so that adjustment costs increase and hence also the market value of capital. Besides, for a given size of the economy, $q^d$ is always higher in case of relatively small governments (thin lines above bold). This is due to the fact that not only an increase in $\sigma$ but also a decrease in the size of the government (smaller $g$) reduce $G_s$, thus rising the corresponding level of $q^d$. If $\sigma = 0$, the initial value of $q$ reduces with $N$ since this rises $G_s$ and with this reduces $q^d$. Independent of the government’s size, $g$, the market value of capital, $q^d$, rises more slowly with $\sigma$ in case of a relatively small region.

Fig. 3(b), (d) and (f) again incorporate the conjunction of production and adjustment cost effect. Due to the complementarity of the inputs, the growth rate is always higher in case of a big government (bold lines above thin lines). This reflects the already discussed argument that more governmental activity increases private capital productivity and with this growth.
However, the results become more sophisticated with respect to institutional policies. If $N = 1$, the growth rate unequivocally increases with $\sigma$. This results because of the externality of private capital accumulation. The individuals perceive the availability of the governmental input to be related stricter to their own capital stock. The results change if $N > 1$. Then the growth rate increases until the above discussed ‘critical value’ $\tilde{N}$ is reached. A further increase in $N$ then induces a decreasing marginal product of capital. Altogether the production effect is positive until $\tilde{N}$ is reached and then becomes negative. While the production effect is ambiguous, the adjustment cost effect unequivocally increases with $\sigma$. Similarly the impact of the degree of congestion can be analyzed. Putting the effects together it is possible to derive a growth maximizing degree of rivalry. This latter is the smaller the higher $N$, since then the adjustment cost effect becomes more and more dominant.

The model has also been calibrated with respect to the parameters in Table 2. Numerical results for the market value of capital and the growth rate under alternative fiscal and institutional arrangements can be found in Table 3. This illustrates the impact of (i) alternative governmental sizes for given $N$ and $\sigma$, or (ii) alternative regional sizes for given $g$ and $\sigma$, or (iii) alternative degrees of congestion for given $g$ and $N$. The shaded values especially clarify that for a
certain given size of the region changes in the type of governmental input may or may not increase growth depending on the prevailing degree of congestion. If we consider e. g. a region that is twice as large as the reference region a transition from the provision of a pure public good to a more congested public input at first spurs growth. Given that $g = 0.0625$, the growth rate increases from $\dot{c} = 0.69287$ to $\dot{c} = 0.73456$; or in case of $g = 0.125$ from $\dot{c} = 1.93804$ to $\dot{c} = 2.08837$ respectively. A further increase in congestion (or the provision of a regional public good) reduces the growth rate. These results hold independent of the government’s size. Hence the instrument that initially is apt to spur growth fails if a critical value of congestion, here $\sigma = 0.70$, is exceeded.

To sum up

The simulation as well as the calibration results make clear that a growth maximizing policy may be unequivocally realized if the government provides more of the complementary production input. However, the result differs with respect to the region’s size and the characteristics of the public production input: While the government in relatively small regions should provide local public inputs, another result applies for relatively big regions. Then governmental production inputs that have the characteristics of pure public goods, e. g. basic research, contribute better to maximizing growth. It is also imaginable that for a certain regional size a transition to a more congested production input at first spurs growth and then reduces it.

6. Conclusions

This paper analyzes the growth impact of fiscal and institutional governmental policies in a regional context. The government provides a productive input that is complementary to private capital. Institutional policies determine the type of governmental input provided as well as the decision on number of firms that are settled, that is the region’s size. Fiscal policies decide on the extent of the public input, and it is assumed that the governmental budget balances each period. Private capital accumulation incurs adjustment costs that depend upon the ratio between private and public investment. Due to the model specification, the governmental input affects private capital productivity thus inducing a production effect. In addition, also adjustment costs are influenced; hence the governmental input also incorporates an adjustment cost effect.

With introducing adjustment costs, an optimal fiscal policy is more complex since the decentralized growth rate is also considerably determined by the market value of capital. The latter is a positive function of the ratio between private and public investment, and with this affects adjustment costs. These incorporate a static dimension (the more private investment the higher are the costs in each period) and a dynamic dimension (investment in one period contributes to an increase in the existing capital stock and with this reduces future investment costs).

After deriving the decentralized equilibrium, fiscal and institutional policies as well as their welfare implications are discussed. Due to congestion externalities, the marginal product of capital as perceived by the individuals exceeds the optimal one whenever congestion arises. Hence, the decentralized and the first–best optimum do not coincide. However, some statements about the optimal ratio between the government and aggregate capital may be derived: It decreases with the region’s size as long as the public production input is partially congested whereas the opposite

<table>
<thead>
<tr>
<th>N=1</th>
<th>$\sigma = 0$</th>
<th>$g=0.0625$</th>
<th>$g=0.125$</th>
<th>$\dot{c} (%)$</th>
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<tr>
<td></td>
<td></td>
<td>1.75024</td>
<td>1.45543</td>
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<th>$g=0.0625$</th>
<th>$g=0.125$</th>
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<tr>
<td></td>
<td>$\sigma = 1.5$</td>
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</table>
applies if the governmental input is a pure public good. Since it is not possible to derive closed-form solutions of the equilibria, optimal tax policy may not be derived explicitly.

We therefore specify the production function and focus on the growth effects of alternative fiscal and institutional policies via carrying out comparative analysis. This is done in the context of a Cobb–Douglas production technology and for two alternative and exogenously given sizes of the government. Several results are derived from simulation and calibration: (i) Bigger governments reduce the market value of capital for all degrees of congestion and for all sizes of the economy. Hence increasing the size of the government unequivocally spurs growth. (ii) In smaller economies the provision of local public inputs goes along with a bigger growth effect than the provision of pure public goods. However, the opposite applies to relatively bigger economies. Then the growth effect of pure public goods exceeds that of regional public goods. (iii) Independent of the size of the economy, the market value of capital increases with rivalry and decreases with the size of the government. (iv) For all sizes of the economy that exceed unity there exists a critical population size that separates the cases in which increasing congestion spurs growth from those that reduce growth. Similarly a critical size of the degree of congestion may be derived.

The discussion clarifies that regional growth strategies have to bear in mind very precisely several facts, among them the relative size of the region and the type of the governmental input. Due to the specification of the production technology and the adjustment costs, the public input has a double positive impact: on the one hand, it increases private capital productivity, and on the other hand, it lowers today’s and future adjustment costs. Both effects raise the growth rate. Basically a regional growth strategy may include several multiple components, namely the decision on the type of the governmental input for a given size of the region or a directed settling policy for a given regional endowment with public goods. In order to increase growth, relatively small regions should provide public inputs with a higher degree of congestion than relatively large regions. Due to the feedback effects within the model both policies may not be determined independently, if the goal to increase growth is to be pursued. It is thus important to coordinate fiscal and institutional policies very carefully.

Appendix A. Relationship between general production function and intensive form

As the production function (1) is assumed to be homogeneous of degree one in the two arguments \( k \) and \( G_s \), Euler’s theorem implies

\[
y = f_k k + f_{G_s} G_s = k \left( f_k + f_{G_s} \frac{G_s}{k} \right). \tag{A.1}
\]

Taking the total differential of (1) leads to

\[
dy = f_k dk + f_{G_s} dG_s = dk \left( f_k + f_{G_s} \frac{dG_s}{dk} \right). \tag{A.2}
\]

In order to characterize the production technology with respect to the interdependence between individual capital stock, \( k \), and public input, \( G_s \), we have to consider the relationships \( g = G/K \) and \( K = Nk \) which are fulfilled in any equilibrium. Hence, the congestion function (2) results to be linear in capital

\[
G_s = gN^{1-\sigma} k \tag{A.3}
\]

and therefore \( G_s/k = dG_s/dk \). Note that the congestion externality drives a wedge between private and social capital return as can be seen in the description of the decentralized economy around Eq. (11). Inserting \( G_s/k = dG_s/dk \) into Eq. (A.2) and combining Eqs. (A.1) and (A.2) yields \( y/k = dy/dk \). So any production function having the above homogeneity properties can be written in the ‘\( Ak \text{–form} \)’

\[
y = z \left( g k^{-(1-\sigma)} K^{(1-\sigma)} \right) k = z \left( gN^{1-\sigma} \right) k. \tag{A.4}
\]
Appendix B. Derivation of (17)

Setting (16) equal to zero leads to the two solutions for the value of capital

\[ q_{1,2} = \frac{1 - \tau_i}{2 - \varepsilon \sigma} \left[ 1 - \varepsilon \sigma - \frac{2b(\varepsilon \rho - (1 - \varepsilon)\delta)}{gN^{1-\sigma}} \right] \pm \sqrt{\left( 1 - \varepsilon \sigma - \frac{2b(\varepsilon \rho - (1 - \varepsilon)\delta)}{gN^{1-\sigma}} \right)^2 + \frac{\varepsilon(2 - \varepsilon \sigma)}{1 - \tau_i} \left( \frac{4b(1 - \tau_i)f_k}{gN^{1-\sigma}} + (1 - \tau_i)\sigma \right)} \right]. \quad (B.1)

We restrict on parameter values which lead to real values of \( q \), that is to positive values of \( \Delta \). Since the second term of \( \Delta \) is positive, \( \sqrt{\Delta} > 1 - \varepsilon \sigma - 2b(\varepsilon \rho - (1 - \varepsilon)\delta)/(gN^{1-\sigma}) \). Hence, \( q_2 \) is negative and the unique solution for the steady state value of capital results in (17).

Appendix C. First–best optimum

The first–best optimum reflects the decisions of the central planner who possesses complete information and chooses all quantities directly, taking into account the congestion caused by all agents and fixing the size of the governmental input. Using \( K=Nk \) and \( g=G/K \), the congestion function (2) modifies to

\[ G_s = gN^{1-\sigma}k \quad (C.1) \]

and with this the planner’s production function is given by

\[ y = f(G_s, k) = \pi(gN^{1-\sigma})k \quad (C.2) \]

The formal optimization is to maximize the agent’s utility (6) subject to (5) and the economy–wide resource constraint

\[ f = c + gk + \iota(1 + \phi). \quad (C.3) \]

The resulting Hamiltonian is similar to (8) and the corresponding first-order conditions imply

\[ c^{-1/\varepsilon} = \lambda \quad (C.4a) \]

\[ 1 + \phi + \iota\phi_i = q \quad (C.4b) \]

\[ \frac{f_k - g}{q} - \frac{\iota\phi_i}{q} + \frac{\dot{q}}{q} - \delta = \rho - \frac{\lambda}{\lambda}. \quad (C.4c) \]

Eq. (C.4b) determines the optimal ratio between private and public investment, \( \frac{\iota}{G_s} \), function of \( q \). Note that the term \( \phi_i \) is also a function of this ratio. It is thus necessary to use the specified function (12) in order to solve (C.4b) for \( \iota/G_s \).

As ratio between private and public investment results

\[ \left( \frac{\iota}{G_s} \right)^* = \frac{q - 1}{2b}. \quad (C.5) \]
An optimum requires that both, the growth rate, \((\hat{c})^*\), as well as the size of the government, \(g^*\), have to be set optimally. The latter is determined by the planner’s optimization problem and leads to the additional optimality condition
\[
f_g - k - \tau \phi_g = 0. \tag{C.6}
\]
Utilizing (C.2), (C.5) and rearranging illustrates that the optimal value of \(g\) may be only determined implicitly as given by
\[
x^* (g^* N^{1-\sigma}) + \frac{(q - 1)^2}{4b} N^{1-\sigma} = 1. \tag{C.7}
\]
Together with \(\phi_k^* = -\phi^* u(G, k)\) and \(G_v/k = g N^{1-\sigma}\) as well as \(\hat{q} = 0\) in the steady state, the first-best growth rate is given by
\[
(\hat{c})^* = \hat{\epsilon} \left[ \frac{f_k - g}{q} + \frac{(q - 1)^2 g N^{1-\sigma}}{q^4 b} - \delta - \rho \right] \tag{C.8}
\]
and inserting (C.7) leads to
\[
\hat{c}^* = \hat{\epsilon} \left[ \frac{x(g^* N^{1-\sigma})(1 - \eta_{yg}^*)}{q^*} - \delta - \rho \right]. \tag{C.9}
\]
It includes the value of capital that may be derived from setting \(\hat{q} = 0\) in (C.4c) and utilizing (C.4a) and (C.7). It is given by
\[
q^* = \frac{x(g^* N^{1-\sigma})(\epsilon (1 - \eta_{yg}^*) + 2\eta^*) - 2g^*(1 - N^{1-\sigma})}{\epsilon \rho - (1 - \epsilon) \delta + 2g^* N^{1-\sigma}}. \tag{C.10}
\]

References


