Taxation, Innovation and Entrepreneurship*

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Abstract

We examine how basic research should be financed. While basic research is a public good benefiting innovating entrepreneurs it also affects the entire economy: occupational choices of potential entrepreneurs, wages of workers, dividends to shareholders, and aggregate output. We show that the general economy impact of basic research rationalizes a pecking order of taxation to finance basic research. In particular, in a society with desirable dense entrepreneurial activity, a large share of funds for basic research should be financed by labor taxation and a minor share is left to profit taxation. Such tax schemes induce a significant share of agents to become entrepreneurs, thereby rationalizing substantial investments in basic research. These entrepreneurial economies, however, may make a majority of citizens worse off if those individuals do not possess shares of final good producers in the economy. In such circumstances, stagnation may prevail.

Keywords Basic research · Economic growth · Entrepreneurship · Income taxation · Political economy
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1 Introduction

Motivation: Financing basic research

Basic research is a public good and has to be provided by the government. The main beneficiaries are innovating entrepreneurs as basic research improves their chances to develop new varieties or new, less cost-intensive production technologies. However, basic research impacts on the entire economy. Specifically, it impacts on:

- the occupational choice of individuals to become entrepreneurs
- wages earned by workers
- dividends paid to shareholders of final good producers
- aggregate output

Financing the public good basic research is therefore an intricate task as taxation will affect the four channels described above and thus it will interact with the incentives to provide basic research. In this paper we examine financing of optimal basic research investments in a general equilibrium framework.

Motivation: Entrepreneurship

The role of entrepreneurship for the well-being of societies has been a constant concern for policy-makers and is at the center of policy debates on how to induce growth in the Eurozone (Economist, 2012). With basic research and taxation, we will examine key drivers that shape entrepreneurial activities in societies. We will analyze in which circumstances societies are in favor of entrepreneurial economies or are prone to remain stagnant.

Model

We develop a simple model of creative destruction where a final consumption good is produced using labor and a continuum of indivisible intermediate goods as inputs. Entrepreneurs can take up basic research provided by the government and invest in applied research in order to develop improved production technologies for intermediates, allowing successful entrepreneurs to earn monopoly profits. In addition, entrepreneurship has immaterial cost (such as entrepreneurial effort cost) and benefits (such as social status). Potential entrepreneurs weigh these benefits against the labor income lost when deciding on whether or not to become entrepreneurs. The government finances its basic research investments using a combination of labor income, profit, and potentially lump-sum taxes. Among others, this financing decision affects the occupational choice by potential entrepreneurs and hence it impacts on the effectiveness of basic research investments.

Main results

Our analyses show that the general economic implications of basic research rationalize a pecking order of taxation to finance basic research. In particular, in an innovative economy with desirable dense entrepreneurial activity, a large share of funds for basic research should be financed by labor taxation and a minor share is left to profit taxation. This pecking order of taxation with labor income tax first exploits the complementarity between basic research and tax policies: The resulting tax stimulus fosters entrepreneurship, thereby increasing benefits from investments in basic research. However, these efficient policies maximizing aggregate output may not be supported by a median voter with little shareholdings if technological progress is labor saving. In this case, a conflict between equality and efficiency may arise. We show that this conflict can be resolved if constitutional bounds on taxation are not too restrictive. As a consequence, while constitutional upper bounds may be introduced with the intention to protect firm owners from expropriation, the general equilibrium effects may imply that the firm owners are actually harmed by the tax bounds: they may undermine political support for an entrepreneurial policy.

The main insights are detailed and qualified in a series of formal results: First, existence and uniqueness of an equilibrium given basic research investments and given tax policies is established. Second, if the government is solely interested in maximizing aggregate consumption, the main insights occur as described, both for a scenario with and without lump-sum taxes. Third, the optimality of a pecking order of taxation survives the use of a broader welfare measure which includes in addition immaterial cost and benefits associated with entrepreneurial activity. Fourth, we characterize the political viability of entrepreneurial policies within a median voter framework.

Organization of the paper

The paper is organized as follows: Section 2 embeds our paper in the literature. Sections 3 and 4 outline the model and derive the equilibrium for given tax policies and basic research investments. In section 5 we analyze aggregate consumption optimal policies for the scenario with and without lump-sum taxes. Section 6 presents an analysis of the political economy of financing basic research. In section 7 we characterize optimal policies for a broader welfare measure that next to aggregate consumption considers immaterial cost and benefits associated with being an entrepreneur. Section 8 concludes.

2 Literature

Our paper is related to several important strands of the literature.

Rationale for public funding of basic research

The case for public funding of basic research is well established in the literature, in particular since the seminal paper of Nelson (1959). He identifies fundamental conflicts between providing basic research and the interests of profit making firms in a competitive economy. First, the provision of basic research has significant positive external effects that cannot be internalized by private firms. Basic research should not be directed towards particular technologies and the resulting scientific knowledge has typically practical value in many fields. As a consequence, technological specialization and a lack of patentability frequently prevent private firms from exploiting all the potential benefits from undirected basic research. Even more, Nelson argues that full and free dissemination of scientific knowledge would be socially desirable due to its non-rivalry. Second, Nelson argues that the long lag between basic research and the reflection thereof in marketable products might prevent short-sighted firms from investing. And third, he points out that the high uncertainty involved in the process might induce a private provision of basic research that is below the socially optimal level. These three problems are the more severe, the more basic the research is and they therefore motivate public provision of basic research in particular.

The case for publicly funded basic research has further been substantiated by several authors. Arrow (1962), for example, points out that invention which he defines as the production of knowledge is prone to three classical reasons for market failures: indivisibility, inappropriability, and uncertainty. Similar to Nelson (1959), he argues that these problems result in an underinvestment in research on the free market and that this problem is the more severe, the more basic the research is. Kay and Smith (1985) stress the enormous benefits from basic research and argue that public provision is necessary due to the public good nature of basic research. They also make a case for domestic provision of basic research rather than free-riding on basic research performed by other countries.

Beginning in the late 1980s, some authors have questioned the public goods nature of scientific knowledge. In particular, the view that existing knowledge is non-rival has been criticized. It has been argued that the utilization of specialized knowledge requires significant investments in complementary research capabilities. This might motivate private provision of basic research (see, for example, Cohen and Levinthal (1989), Rosenberg (1990), or Callon (1994)). Notwithstanding, these authors do not question public provision of basic research. Callon points out that public engagement in the field of science is needed in order to preserve variety and flexibility in the economy.

In summary, there is a strong case for publicly funded research, in particular basic research. This rationale is matched by empirical evidence. Gersbach et al. (2013) report data showing that for a selection of 15 countries the average share of basic research that was performed in the government and higher education sector was approximately 75% in 2009. From the OECD main science and technology indicators we find that across OECD member countries around 80% of total research performed in the government or higher education sector is also funded by the government.[1] Taken together, these findings suggest that indeed a major share of basic research investments are publicly funded.

Financing of basic research

In this paper, we start from the rationale why basic research has to be funded publicly, in particular because pure private provision will result in underinvestment when compared to the social optimum. Our main question is then how optimally chosen basic research expenditures should be financed. Our paper is thus related to the literature on financing productive government expenditures. In the seminal paper, Barro (1990) examines the case of productive government expenditures as a flow variable. Futagami et al. (1993) develop the case of productive government expenditures as a stock variable. In both cases, the public service provided is not subject to congestion effects, as for

[1] Data have been downloaded from OECD (2012) in May 2012. As far as available, 2008 data has been used. For each country, the share of public funding in the government and higher education sector has been computed as follows: $\frac{\text{sub-total government funding in higher education sector + sub-total government funding in government sector}}{\text{total funding higher education sector + total funding government sector}}$. The average of these shares across all OECD member countries was found to be slightly below 80%.

publicly provided basic research. These authors develop investment-based endogenous growth models where the individual firm faces constant returns to scale with respect to both, private capital and the public services provided by the government. According to the comprehensive survey by Irmen and Kuehnel (2009), this applies more general to the main body of the literature on productive government expenditures and economic growth. By contrast, our model is rooted in the tradition of R&D based endogenous growth models, and particularly those that explicitly take into account the hierarchical order of basic and applied research (see, for example, Arnold (1997), Morales (2004), or Gersbach et al. (2010)). In these kind of models, basic research has no productive use in itself, but rather fuels into the productivity of the applied research sector, where knowledge is transformed into blueprints for new or improved products. In our case, basic research affects the innovation probability of entrepreneurs that engage in applied research. Using more public funds for basic research improves the chances of success of private entrepreneurs at the cost of diverting resources away from intermediate and final good production.

Moreover, a second important role of financing basic research will be addressed in this paper. Basic research may be financed via a combination of labor income, profit, and lump-sum taxes. The relative size of labor to profit taxes affects the trade-off faced by potential entrepreneurs between being employed in the labor market and becoming an entrepreneur and hence influences the number of innovating entrepreneurs in our economy.

Tax structures and entrepreneurial activity

Empirical evidence in the literature suggests that the tax structure does indeed influence the level of entrepreneurial activities in an economy. Using cross-sectional data of US personal income tax returns, Cullen and Gordon (2007) estimate the impact of various tax measures on entrepreneurial risk-taking as proxied by an indicator variable for whether or not an individual reports business losses greater than 10% of reported wage income. They find that a cut in personal income tax rates significantly reduces entrepreneurial risk taking. The evidence for a cut in corporate tax rates is less clear: depending on the model specification used, such a cut is predicted to either rise or not to significantly affect entrepreneurial risk taking. Cullen and Gordon interpret their results to be in line with their theory, as risk-sharing of non-diversifiable entrepreneurial risks with the government is positively related to the corporate income tax rate. Djankov et al. (2010) analyze cross-country data for 85 countries. They find that higher effective tax rates paid by a hypothetical new company have a significant adverse effect on aggregate investment and entrepreneurship. Gentry and Hubbard (2000) analyze 1979 to 1992 data from the Panel Study on Income Dynamics to find that less progressive tax rates significantly increase entrepreneurship.

Optimal taxation in an economy with entrepreneurship

We analyze the optimal mix of basic research and tax policies. Hence, our paper is also related to the literature on optimal income taxation standing in the tradition of the seminal paper by Mirrlees (1971). Initial work in this area analyzed tax distortions of the labor leisure choice by households and derived optimal tax policies balancing the efficiency losses from more progressive taxes against welfare gains associated with more egalitarian income distributions (see also Sheshinski (1972) or Stern (1976), for example).

At the heart of our model is the occupational choice by (potential) entrepreneurs. Hence, our paper is closer related to papers with endogenous wages and occupational choice. Feldstein (1973), Allen (1982), and Stiglitz (1982) develop models with two types of workers, skilled and unskilled, with endogenous wages and endogenous labor supply by both types of labor but no occupational choice.[2] Boadway et al. (1991) present a model with heterogeneous agents who can chose between becoming entrepreneurs or workers. These papers have in common that they analyze optimal taxes, where tax rates are the same for both types of labor or income. By contrast, in our model the government can discriminate between taxes on profits and on labor income. Kanbur (1981) considers a model with endogenous occupational choice of homogeneous agents between becoming a worker earning a safe wage or an entrepreneur earning risky profits. Among others, he considers entrepreneurial risk-taking given occupational dependent taxation, but he does not derive optimal tax policies. In this regard his work is close in nature to recent work on calibrated dynamic general equilibrium models that are used to assess the effects of stylized tax reforms (see Meh (2005) or Cagetti and De Nardi (2009), for example).

Moresi (1998) and Scheuer (2011), for example, analyze optimal tax policies in models

[2] In the paper by Allen (1982), workers belong to either of two skill groups and they can chose among two types of labor, but workers perfectly select into these types of work on the basis of their skill-group.

of asymmetric information with occupational choice, where the government faces a trade-off between efficiency and equity. The distinctive feature of our model is that we analyze optimal tax policies in the presence of basic research that allows the government to use tax revenues in order to directly foster innovativeness of entrepreneurs. We show that efficient policies make use of a pecking order of taxation. In particular, in our model investments in basic research that allow for efficiency gains in aggregate should be accompanied by low profit taxes and high labor income taxes. We consider distributional effects when analyzing the political economics of our results.

3 The Model

The economy is populated by a continuum of measure $\bar{L} > 1$ of households who derive utility $u(c) = c$ from a final consumption good. Agents are indexed by k ($k \in [0, \bar{L}]$).

3.1 Production

The final good, denoted by y, is produced with a continuum of intermediate goods $x(i)$ ($i \in [0, 1]$). The production technology is given by

$$y = L_y^{1-\alpha} \int_0^1 x(i)^\alpha \, di \ , \tag{1}$$

where L_y denotes labor employed in final good production and where $0 < \alpha < 1$. The final good is only used for consumption, hence in equilibrium, output of the final good equals aggregate consumption ($C = y$).

We assume that intermediate goods x are indivisible, i.e. $x(i)$ is either 1 or 0.[3] The final consumption good is chosen as the numéraire whose price is normalized to 1.

[3]As we will explain in more detail later on, we consider the case of labor saving technological progress in the intermediate sector. With indivisible intermediate goods, labor saved in intermediates production is not taken up elsewhere in the economy at constant wages. This can give rise to a stark conflict of interest between equality and efficiency in our economy and hence to interesting political economy effects. We discuss these in detail in section 6. Three remarks are in order: First, our main finding of the optimality of a pecking order of taxation does neither rely on labor saving technological innovation nor on the indivisibility of intermediates. It rather follows from the complementarity of basic research and the occupational choice of potential entrepreneurs. Second, we believe the conflict between equality and efficiency in our economy to be broadly in line with the decreasing shares of labor income, in particular for low skilled labor, in aggregate income that can be observed in the recent past in the EU and the US (cf. footnote 25). And third, while the indivisibility of intermediates can accentuate the equality-efficiency trade-off in our economy, it is not necessary for such effects (cf. footnote 26).

Firms in the final good sector operate under perfect competition. They take the price $p(i)$ of intermediate goods as given. In the following we work with a representative final good firm maximizing

$$\pi_y = y - \int_0^1 p(i)x(i)\,di - wL_y \tag{2}$$

by choosing the quantities $x(i) \in \{0,1\}$ and the amount of labor L_y. If the final good producer chooses $x(i) = 1$ for all i, the demand for labor in final good production will be

$$L_y = \left(\frac{1-\alpha}{w}\right)^{\frac{1}{\alpha}}. \tag{3}$$

3.2 Behavior of intermediate good producers

Each intermediate good can be produced by a given technology using $m > 0$ units of labor. Hence the marginal production costs are mw and we assume that the standard technology is freely available. This implies perfect competition and a price equal to the marginal costs. If an entrepreneur engages in research and development and successfully innovates, the production costs decline by a factor γ ($\gamma < 1$) leading to marginal production costs of γmw. The innovating entrepreneur obtains a monopoly and it will turn out that he still offers his product at the price equal to the marginal cost of potential competitors, mw, thereby gaining profit $\pi_{xm} = (1-\gamma)mw$.

3.3 Innovation

There is a measure 1 of individuals $[0,1] \subset [0,\bar{L}]$ who are potential entrepreneurs. Individuals face different costs and benefits when deciding to become an entrepreneur. Specifically, we assume that agents are ordered in $[0,1]$ according to their immaterial utilities from entrepreneurial activities: In particular, individual k faces the utility factor $\lambda_k = (1-k)b$ ($k \in [0,1]$, b being a positive parameter). This factor rescales the profit earned from entrepreneurial activities in order to take into account immaterial cost (such as cost from exerting efforts as an entrepreneur or utility cost from entrepreneurial risk taking that are not reflected in the utility from consumption) and immaterial benefits (such as excitement, initiative taking, or social status) associated with entrepreneurial activity.[4] Agents with a higher index k have lower utility factors.

[4]Cf. footnote 14 for a discussion on how differences in risk-attitudes might give rise to occupational choice effects similar to the ones arising from our immaterial benefit factor λ_k. In our model, there is

A utility factor $\lambda_k < 1$ represents net utility cost from being an entrepreneur while a factor $\lambda_k > 1$ represents net immaterial benefits.[5] For individuals k with $\lambda_k = 1$, and thus $k^{crit} = 1 - \frac{1}{b}$, immaterial cost and benefits associated with entrepreneurial activities cancel out. If b is small and thus k^{crit} is small or even zero, the society is characterized by a population of potential entrepreneurs for whom effort cost matter most. If b is large and thus k^{crit} is large, the potential entrepreneurs enjoy being one compared to a worker. We assume that λ_k is private information and thus only observed by agent k.[6]

The chances of entrepreneurs to successfully innovate can be fostered by basic research. Basic research generates knowledge that is taken up by entrepreneurs and transformed into innovations applied in the production process. Suppose that the government employs L_B ($0 \leq L_B \leq \bar{L}$) researchers in basic research. Then the probability that an entrepreneur successfully innovates is given by $\eta(L_B)$ where $\eta(L_B)$ fulfills $\eta(0) \geq 0$, $\eta'(\cdot) > 0$, $\eta''(\cdot) < 0$ and $\eta(\bar{L}) \leq 1$.[7] Depending on whether $\eta(0) = 0$ or $\eta(0) > 0$, basic research is a necessary condition for innovation or not.

Accordingly, if a measure L_E of the population decided to become entrepreneurs and each has the success probability $\eta(L_B)$, the share of intermediate sectors with successful innovation is equal to $\eta(L_B)L_E$.[8] We note that the property $L_E \leq 1$ allows that entrepreneurs perform research on a variety different from others.[9]

3.4 Financing scheme

Expenditures for basic research have to be financed by taxes. The government can levy taxes on labor income or profits. Additionally, we assume that the government

no aggregate risk and entrepreneurship is not prone to moral hazard. Hence, with complete markets entrepreneurs can perfectly insure against entrepreneurial risk. In this case, λ_k does not capture utility cost from entrepreneurial risk taking.

[5] Our concept of immaterial utilities associated with being an entrepreneur is in line with recent empirical evidence (cf. Hamilton (2000); Benz and Frey (2008); Benz (2009); Douglas and Shepherd (2000); Praag and Versloot (2007)). Most studies find that entrepreneurship involves positive non-monetary benefits.

[6] This does preclude to condition taxation on λ_k. We note that our results remain unaffected if λ_k is common knowledge but tax policies do not condition thereon.

[7] $\eta'(\cdot)$ and $\eta''(\cdot)$ denote the first and second derivative, respectively, of $\eta(\cdot)$ with respect to L_B.

[8] We use a suitable version of the law of large numbers for a continuum of random variables.

[9] Strictly speaking we assume that there is no duplication of research efforts. It is straightforward to incorporate formulations in which several researchers compete for innovation on one variety. This would decrease the benefits from basic research for entrepreneurs and for the society.

can levy lump sum taxes or make lump sum transfers.[10] Later we examine the case when this is not possible. A tax scheme is a vector (t_L, t_P, t_H) where t_L and t_P are the tax rates on labor income and on profits, respectively, and t_H denotes a lump sum tax or transfer. We assume that there are upper bounds (and potentially lower bounds) of labor income and profit taxes given by the constitution. For example in Germany, average taxes on income cannot exceed 50%.[11] We denote the upper and lower bounds by $\bar{t}_j$ and $\underline{t}_j$, $j \in \{L, P\}$, respectively. For our theoretical analysis we assume the upper bounds are strictly smaller than 1, i.e. $\bar{t}_j \leq 1 - \varepsilon$ for some arbitrarily small $\varepsilon > 0$.

Throughout our paper, we assume that the government needs to run a balanced budget, i.e. the government budget constraint is given by

$$wL_B = t_L(\bar{L} - L_E)w + t_P\left(\pi_y + \eta(L_B)L_E\pi_{xm}\right) + t_H\bar{L} \ , \tag{4}$$

where $t_H = 0$ in the scenario without lump-sum taxes.

3.5 Sequence of events

We summarize the sequence of events as follows.

(1) The government hires a number L_B of researchers to provide public basic research and chooses a financing scheme.

(2) A share L_E of the population decide to become entrepreneurs. With probability $\eta(L_B)$ they successfully innovate, which enables them to capture monopoly rents. A share $(1 - \eta(L_B))L_E$ will not be successful and will earn zero profits.

[10] Our model allows for unsuccessful entrepreneurs which earn zero profits. Consequently, in case that their share of the profits of the final good firm are not too high, they may not be able to pay the lump sum tax. For a broad range of parameter values, lump-sum taxes are negative in optimum, implying that this is not an issue. If not, we assume that all individuals have a certain endowment, which could be drawn on by the government in this case.

[11] Alternatively, upper bounds on tax rates may implicitly arise from harmful supply-side effects of taxation: Supply effects of profit taxes are at the very heart of the analysis pursued here. Yet, in an open economy the government might also be confronted with additional harmful supply effects associated with high profit taxes that are not considered here and that may give rise to effective upper bounds on profit taxes. Similarly, supply effects of labor income taxes are only considered to the extent to which they affect the occupational choice by potential entrepreneurs. In addition, labor income taxes might affect the labor/leisure choice of workers and might hence be effectively bound from above. Lower bounds on profit taxes, in particular, might be demanded by the international community. The European Council of Economics and Finance Ministers, for example, has agreed upon a code of conduct for business taxation which is intended to tackle harmful competition in the field of business taxation (European Union, 1998). Although this code of conduct does not define explicit lower bounds on taxation and is not legally binding, it still represents a considerable political commitment not to have extremely low tax rates on profits.

(3) Each intermediate good firm i hires a number $L_x(i)$ of workers in order to produce the intermediate good $x(i)$.

(4) The representative final good firm buys the intermediate goods $x(i)$ at a price $p(i)$ and produces the homogeneous final good y.

4 Equilibrium

In this section we derive the equilibrium for a given amount of basic research and a given financing scheme.

4.1 Occupational choice by potential entrepreneurs

We first address the choice of occupation. Potential entrepreneurs, i.e. agents in the interval $[0, 1]$, can choose between being employed as workers and trying to develop an innovation to be used in the production of intermediate goods. We are left with two cases: all agents choose to be workers or both occupations are chosen in equilibrium.[12] If both occupations are chosen in equilibrium, the marginal entrepreneur has to be indifferent between being employed as a worker and becoming an entrepreneur. The expected net profit of an entrepreneur is

$$\pi^E = (1 - t_P)\eta(L_B)\pi_{xm} = (1 - t_P)w\eta(L_B)m(1 - \gamma) \ .$$

The last expression indicates that the expected profit of the entrepreneur consists of the expected amount of labor saved in intermediate good production, $\chi(L_B) \equiv \eta(L_B)m(1-\gamma)$, scaled by the wage rate net of profit taxes. Hence, the expected utility for an individual k with (dis-)utility factor $\lambda_k = (1 - k)b$ from being an entrepreneur is[13]

$$EU^E(k) = (1 - t_P)w\chi(L_B)(1 - k)b \ .$$

[12] More precisely, in the first case only a set of individuals of measure 0 decides to become an entrepreneur.

[13] We note that we have chosen a multiplicative functional form. An alternative approach is to use an additive functional form by deducting the cost (see e.g. Boadway et al. (1991) or Scheuer (2011)). The multiplicative approach is more convenient and analytically much easier. In addition, it implies that the net immaterial benefit is scaled by entrepreneurial profits. The multiplicative approach may therefore be more appropriate to reflect effort costs and social status benefits, in particular, as these would typically be related to profits. For $\lambda_k < 1$ the effort cost dominate, while for $\lambda_k > 1$ the social status benefits dominate. Qualitatively, however, the additive and the multiplicative approach involve the same trade-offs and pecking order considerations.

The individual is indifferent between being employed as a worker and becoming an entrepreneur if $EU^E(k) = (1-t_L)w$. Solving for the equilibrium amount of entrepreneurs yields[14]

$$L_E^e = \max\left\{0; 1 - \frac{1-t_L}{1-t_P}\frac{1}{\chi(L_B)b}\right\} . \tag{5}$$

In the following, we use τ as an abbreviation for $\frac{1-t_P}{1-t_L}$, with the upper and lower bounds of τ denoted by $\overline{\tau}$ and $\underline{\tau}$ being defined by the respective bounds of t_L and t_P. τ is a measure of tax incentives given to (potential) entrepreneurs. Moreover, let $\overline{\tau} \geq 1 \geq \underline{\tau}$ implying that a neutral tax policy $t_L = t_P$ is always possible.

Knowing L_E from (5), we obtain the amount of labor employed in the production of intermediates as

$$L_x^e = \int_0^1 L_x(i)di = m - \chi(L_B)L_E^e \tag{6}$$

if $x(i) = 1 \ \forall \ i$. This corresponds to the amount of labor necessary to produce the intermediate goods with the old technology less the (expected) amount of labor saved by the new technologies invented by the entrepreneurs.

4.2 Equilibrium for given basic research and financing scheme

We will now derive the equilibrium for given basic research and tax policy. Due to the indivisibility of the different varieties of the intermediate goods, we have to consider the case that despite diminishing returns to intermediate goods in final good production, the final good firm will not use all of the different varieties or will even go out of

[14] In our model, potential entrepreneurs differ in their immaterial cost and benefits from being an entrepreneur. Agents whose expected utility from being an entrepreneur exceeds the utility from working in the labor market opt to become an entrepreneur, thus giving rise to continuous occupational choice effects. We note that a similar result for the occupational choice would arise if agents differed in the risk attitude rather than in an extra (dis-)utility term. Suppose for example that potential entrepreneurs differ only in their degree of constant relative risk-aversion with $u_k(c) = \frac{c^{(1-r_k)}}{1-r_k}$, where r_k is distributed according to some continuous and differentiable distribution function $F_{r_k}(r_k)$ on $[0,1]$, satisfying $\frac{dF_{r_k}(\cdot)}{dr_k} > 0, \ \forall r_k \in [0,1]$. Suppose further that insurance against entrepreneurial risks is not possible. Then, individual k opts to become an entrepreneur if his certainty equivalent from being an entrepreneur is at least as large as his after-tax wage: $[\eta(L_B)]^{\frac{1}{1-r_k}}(1-t_P)m(1-\gamma)w \geq (1-t_L)w$ for the case of no other income. It follows that all potential entrepreneurs with $r_k \leq \bar{r} = \max\left\{0; 1 - \frac{\ln(\eta(L_B))}{\ln(1-t_L)-\ln((1-t_P)m(1-\gamma))}\right\}$ will become entrepreneurs. The equilibrium number of entrepreneurs is then given by $L_E = F_{r_k}(\bar{r})$. As for the case with heterogeneous immaterial cost and benefits from being entrepreneur, entrepreneurship is increasing in m, t_L, and L_B, decreasing in t_P and γ and it is independent of w. However, basic research has an additional effect here: next to increasing the expected profit from being an entrepreneur, it affects associated entrepreneurial risks.

business and not produce at all. We start by considering the equilibrium in the market for intermediate goods with positive production in the final good sector:

Lemma 1

(i) In any equilibrium with positive production in the final good sector, intermediate good producers supplying their product will charge $p_i = mw$.[15]

(ii) In any equilibrium with positive production in the final good sector, the final good producer uses all varieties of intermediate goods.

The proof of Lemma 1 can be found in Appendix A.1. As a consequence of point (ii) of Lemma 1, we can use the equilibrium number of entrepreneurs in (5) and labor in intermediate good production in (6) together with the market clearing condition in the labor market

$$\bar{L} = L_E + L_B + L_y + L_x \ , \tag{7}$$

to derive the number of workers employed in the final good sector in an equilibrium with positive final good production:

$$L_y^e = \bar{L} - L_B - L_E^e - L_x^e \ . \tag{8}$$

Equation (3) yields the corresponding equilibrium wage rate as

$$w^e = (1-\alpha)(\bar{L} - L_B - L_E^e - L_x^e)^{-\alpha} \ . \tag{9}$$

Finally we determine when an equilibrium with positive production will occur, that is, under which condition the final good firm will make positive profits. Using the profit function (2) and Lemma 1, we obtain in equilibrium

$$\pi_y^e = (L_y^e)^{1-\alpha} - w^e L_y^e - w^e m \ .$$

Inserting the equilibrium wage rate (9) yields:

$$\pi_y^e = \alpha(L_y^e)^{1-\alpha} - (1-\alpha)m(L_y^e)^{-\alpha}.$$

We observe that the final good firm's profit strictly increases in the amount of labor it employs in equilibrium. This is very intuitive as higher employment in final good

[15] To avoid the need to discretize the strategy-space in order to obtain existence of equilibria in the price-setting game in the intermediate good industry i, we assume as a tie-breaking rule that the final good producer demands the product from the innovating entrepreneur if he offers the same price as non-innovating competitors.

production yields higher output and in addition, the final good producer increases employment if wages are lower, implying that the prices of both inputs labor and intermediate goods are lower. Consequently, the final good firm's profits will be positive, if the amount of labor employed in final good production exceeds the critical level, $L_y^c \equiv m\frac{1-\alpha}{\alpha}$. By (8), this will always be the case in equilibrium, if governmental policy (L_B, τ) satisfies the following condition

$$\frac{m}{\alpha} \leq \begin{cases} \bar{L} - L_B & \text{if} \quad \frac{1}{\tau\chi(L_B)b} \geq 1 \\ \bar{L} - L_B + \left[1 - \frac{1}{\tau\chi(L_B)b}\right][\chi(L_B) - 1] & \text{if} \quad \frac{1}{\tau\chi(L_B)b} < 1 \end{cases}. \tag{PPC}$$

Otherwise the wage rate is too high such that the indivisible intermediate goods are too expensive to realize positive profits.

We are now in a position to characterize the allocation and prices in the equilibrium of the economy for given basic research investments L_B and a given financing scheme τ.

Proposition 1

(i) If L_B and τ satisfy condition (PPC), *there is a unique equilibrium with $x(i) = 1$ for all i and*

(1) $L_E^e = \max\left\{0; 1 - \frac{1-t_L}{1-t_P}\frac{1}{\chi(L_B)b}\right\}$

(2) $L_x^e = m - \chi(L_B)L_E^e$

(3) $L_y^e = \bar{L} - L_B - m + L_E^e[\chi(L_B) - 1]$

(4) $w^e = (1-\alpha)\left(L_y^e\right)^{-\alpha}$

(5) $p(i)^e = m(1-\alpha)\left(L_y^e\right)^{-\alpha} \; \forall i$

(6) $y^e = \left(L_y^e\right)^{1-\alpha}$

(7) $\pi_y^e = \left(L_y^e\right)^{-\alpha}\left(\alpha L_y^e - m(1-\alpha)\right)$

(8) $\pi_{xm}^e = (1-\gamma)m(1-\alpha)(L_y^e)^{-\alpha}$.

(ii) If L_B and τ do not satisfy condition (PPC), *there is a unique equilibrium with $x(i) = 0$ for all i, $L_E^e = L_x^e = L_y^e = 0$, and zero profits.*

The proof of Proposition 1 can be found in appendix A.2.

5 Optimal Policies

The government can manipulate the previously established equilibrium outcomes by investing in basic research and via the tax scheme. The government's objective is to maximize welfare of the economy, which comprises a material component, consumption, and an immaterial component, the entrepreneurs' (dis-)utility of being an entrepreneur. The utility of being an entrepreneur cannot be observed directly by the government. Still, simple model framework the government can determine the immaterial welfare component from the revealed occupational choices of the individuals together with the precise distribution of disutilities from being entrepreneur. As this distribution may be impossible to observe in reality, we first consider a government that concentrates on the material welfare component, that is on aggregate consumption. Later in section 7 we will take the viewpoint of a government that maximizes aggregate welfare, accounting in addition for the utility costs and benefits from becoming an entrepreneur. There, we will show that our main insight regarding the pecking order of taxation prevails and may be reinforced with a broader welfare measure. We will now start our discussion of optimal policies with some preliminary considerations before we turn to the solutions of the government's maximization problem.

5.1 Preliminary considerations

Efficient vs. inefficient entrepreneurship

Note that before taxes, the expected profit of an entrepreneur is higher than the wage rate in goods production if $\chi(L_B) \geq 1$. That is, by the entrepreneurial activity the individual saves more labor in intermediate good production than the unit of labor he could provide himself to the labor market. However, even if entrepreneurship would have a negative impact on labor supply in final good production, i.e. if $\chi(L_B) < 1$, individuals may find it worthwhile to become an entrepreneur due to immaterial benefits and the tax policy τ. In this respect, we make the following assumption:

Assumption 1

(i) $\chi(0) < 1$, *(ii)* $1/\bar{\tau} < b \leq 1/\chi(0)$.

Assumption 1(i) states that, in expectation, entrepreneurship will reduce the labor supply for final good production, and thus final output, when no basic research is provided. The last inequality in the second condition allows the government to preclude output reducing entrepreneurship by implementing a neutral tax policy and not investing in basic research. By contrast, the first inequality ensures that in the situation with labor-saving entrepreneurship, the government will be able to induce a positive measure of individuals to become entrepreneurs via its tax policy.

Positive production in final good sector

In setting its policy (L_B, τ), the government has to consider condition (PPC) which determines the resulting equilibrium type. The following assumption ensures that any aggregate consumption optimal policy will yield an equilibrium with positive final good production and that we can neglect (PPC) in the government's optimization problem.

Assumption 2
$\bar{L} \geq \frac{m}{\alpha}$,

As we will show at the beginning of the next section, the aim of the government's basic research and tax policies boils down to maximizing the amount of labor available for final good production. Hence, if some feasible policy choice satisfies condition (PPC), then so does the optimal policy choice. By Assumption 1(ii), the government can fully suppress entrepreneurship by choosing $L_B = 0$ and $\tau = 1$. Assumption 2 ensures that final good producers' profits are non-negative under this policy regime and hence so they are under the aggregate consumption optimal policy regime.

We now derive optimal basic research policy when lump-sum taxes and transfers are available to the government. As the number of entrepreneurs only depends on the relation between profit and labor income taxes as captured by τ, the assumption of lump-sum transfers allows us to separate the choice of L_B from the choice of the government's tax incentives to (potential) entrepreneurs. This scenario will yield the major insights.[16] If no lump sum taxes and transfers are available, the choices of τ and L_B cannot in all cases be separated. We discuss these issues in section 5.3 and abstract from such problems in the next section.

5.2 Optimal policy with lump-sum taxes and transfers

The government's problem of maximizing material welfare boils down to maximizing aggregate consumption, C, by choosing the amount of basic research, L_B, and the optimal ratio between profit and labor taxes, τ, while either levying an additional lump sum tax if labor and profit taxes satisfying the optimal τ do not suffice to finance

[16] Given that basic research investments account for a share of government expenditures only, the scenario with lump-sum taxes might also be interpreted as one where any excess funds are used to finance other government expenditures that benefit all members of the population equally. For a broad range of parameter values, lump-sum taxes are negative in optimum, i.e. we have lump-sum transfers. Then, our analysis is equivalent to an analysis with no lump-sum taxes but investments in an additional public good g which directly impacts on households' utilities and where $u(c, g) = c + \frac{g}{L}$.

the desired amount of L_B or making a lump sum transfer in case that the revenue generated by τ is larger than needed for the basic research expenditures.

$$\begin{aligned}\max_{\{t_L,t_P,t_H,L_B\}} \quad C =& \pi_y + \eta(L_B)L_E\pi_{xm} + wL_y + wL_x + wL_B - (\bar{L} - L_E)wt_L \\ & - t_P\left[\pi_y + \eta(L_B)L_E\pi_{xm}\right] - t_H\bar{L} \\ \text{s.t.} \quad wL_B =& (\bar{L} - L_E)wt_L + t_P\left[\pi_y + \eta(L_B)L_E\pi_{xm}\right] + t_H\bar{L}\end{aligned}$$

where $t_H\bar{L}$ denotes the lump-sum taxes or transfers. Inserting the constraint into the objective function and using the aggregate income identity $y = \pi_y + \eta(L_B)L_E\pi_{xm} + wL_y + wL_x$ reduces the problem to

$$\begin{aligned}\max_{\{L_B,\tau\}} \quad C(L_B,\tau) = y(L_B,\tau) &= (L_y^e(L_B,\tau))^{1-\alpha} \\ &= \left[\bar{L} - L_E(L_B,\tau) - L_B - L_x(L_B,\tau)\right]^{1-\alpha} \ .\end{aligned}$$

Hence, the objective of the government is to maximize the amount of productive labor in final good production. By inserting L_x, the objective function can be written as:

$$\left[\bar{L} - L_B - m + L_E[\chi(L_B) - 1]\right]^{1-\alpha} \ . \tag{10}$$

Maximization of (10) is equivalent to maximization of $\bar{L} - L_B - m + L_E[\chi(L_B) - 1]$ which we will use in the following.

It will be informative to solve the government's problem in two steps. First, we determine the optimal tax policy to finance a given amount of basic research. In the second step, we use the optimal tax policy to derive the optimal basic research investments. In the optimization at the first step, the Kuhn-Tucker conditions with respect to the optimal tax policy are:

$$\frac{\partial L_E}{\partial \tau}[\chi(L_B) - 1] \gtreqless 0, \tag{11a}$$

$$\frac{\partial L_y}{\partial \tau}(\tau - \underline{\tau})(\overline{\tau} - \tau) = 0 \ . \tag{11b}$$

The term in brackets on the left-hand side of (11a) expresses how much labor in intermediate-good production will be saved in expectation by an additional entrepreneur. We also observe in (11a) that the expected benefit of another entrepreneur depends on the level of basic research expenditures. For example, if $\eta(0) \approx 0$ implying $\chi(0) \approx 0$, an entrepreneur is not as productive in innovating than when working in final good production. Only if the amount of basic research is larger than $L_{B,min} \equiv \max\{0, \eta^{-1}(1/[m(1-\gamma)])\}$, where $\eta^{-1}(\cdot)$ denotes the inverse of $\eta(\cdot)$, will an increase in entrepreneurship be favorable for aggregate consumption.[17] Note that $\frac{\partial L_E}{\partial \tau}$ is clearly non-negative and with $L_B \geq L_{B,min}$ strictly positive for τ in the neighborhood of $\bar{\tau}$ according to Assumption 1. Consequently, if $L_B > L_{B,min}$, the government will increase τ to its maximum to make entrepreneurship most attractive. The opposite is the case if $L_B < L_{B,min}$. Then the government aims at reducing the number of entrepreneurs to a minimum by setting τ to its lowest level.[18] The government's tax policy is indeterminate when $L_B = L_{B,min}$ and we assume that it sets $\tau = \bar{\tau}$ in this case. We summarize our finding in the next proposition.

Proposition 2 (Optimal tax policy)
For a given amount of basic research, L_B, the government levies taxes such that

$$\tau = \begin{cases} \bar{\tau} & \text{if } L_B \geq L_{B,min} \\ \underline{\tau} & \text{if } L_B < L_{B,min} \end{cases} \tag{12}$$

We will now determine the optimal basic research investments in the second step of the government's optimization problem. Given Lemma 2, we can split the maximization problem at the second step into one where L_B is constrained on $L_B \geq L_{B,min}$ and another for $L_B < L_{B,min}$. Regarding the first problem

$$\max_{\{L_B \geq L_{B,min}\}} \quad C(L_B, \tau) = y(L_B, \tau) \ ,$$

$$\text{s.t.} \quad \tau = \bar{\tau} \ ,$$

we obtain the necessary conditions for a maximum

$$\frac{\partial L_E(L_B, \bar{\tau})}{\partial L_B}[\chi(L_B) - 1] + L_E(L_B, \bar{\tau})\chi'(L_B) - 1 \leq 0 \ , \tag{13a}$$

$$\frac{\partial L_y(L_B, \bar{\tau})}{\partial L_B}(L_B - L_{B,min}) = 0 \ . \tag{13b}$$

[17]Note that $L_{B,min}$ is positive by Assumption 1(i) stating that without basic research the entrepreneurs are not as productive in producing labor saving innovations as in working in final good production. This assumption is not necessary for our results in section 5. With $\chi(0) \geq 1$, the government would always choose a tax policy $\tau = \bar{\tau}$ and basic research investments, if positive, will further increase the number of entrepreneurs. The latter is due to the fact that by our specification of the immaterial utility component of entrepreneurship, the corner solution $L_E = 1$ is precluded.

[18]Note that for $L_B < L_{B,min}$, there are typically multiple tax policies that entirely discourage entrepreneurship. For instance, by Assumption 1(ii), for $L_B = 0$ the government is indifferent between any tax policies (t_L, t_P) satisfying $\tau \in [\underline{\tau}, 1]$. For simplicity, we assume that the government implements $\underline{\tau}$, i.e. $t_L = \bar{t}_L$, $t_P = \underline{t}_P$ in such cases.

Marginally increasing basic research investments has three different effects on final good production: First, it improves the innovation prospects of "old" entrepreneurs as reflected by the second term in equation (13a).[19] Second, the increase in innovation prospects attracts additional entrepreneurs as reflected in the first term of equation (13a). Note that by $L_B \geq L_{B,min}$ (and hence $\chi(L_B) \geq 1$), this rise in entrepreneurship is beneficial for final good production. The optimal choice of L_B trades-off these gains from investments in basic research against the loss of the marginal unit of labor used in basic research rather than in final good production. This marginal labor cost of basic research is reflected by the term -1 in equation (13a). Let us denote the solution of this constrained maximization problem by $\tilde{L}_B(\bar{\tau})$. Note that if $\tilde{L}_B(\bar{\tau}) > L_{B,min}$, it will satisfy (13a) with equality.

With respect to the maximization problem constrained by $L_B < L_{B,min}$, which implies tax policy $\tau = \underline{\tau}$, we can directly infer that the solution will be $\tilde{L}_B(\underline{\tau}) = 0$. The reason is that basic research affects consumption only by improving the success probabilities of entrepreneurs. However, for all $L_B < L_{B,min}$ entrepreneurship negatively affects final output and by Assumption 1 the government is able to deter such inefficient entrepreneurship by not providing basic research.

Consequently, the government decides between implementing the policies $(\tilde{L}_B(\bar{\tau}), \bar{\tau})$ or $(0, \underline{\tau})$. In the first situation with positive basic research and entrepreneurship, we speak of an *entrepreneurial* economy and refer to the second situation without basic research investments and entrepreneurship as a *stagnant* economy. The government implements the policy with positive basic research investments and a tax policy favoring entrepreneurship if and only if this leads to higher labor supply in final good production and hence higher consumption vis-à-vis the stagnant economy. In the stagnant economy, labor supply for final good production is given by $L_y = \bar{L} - m$. Hence, we observe from Proposition 1 that the government opts for the entrepreneurial economy if and only if it satisfies the following condition:

$$-\tilde{L}_B(\bar{\tau}) + \left[1 - \frac{1}{\bar{\tau} b \chi\left(\tilde{L}_B(\bar{\tau})\right)}\right] \left[\chi\left(\tilde{L}_B(\bar{\tau})\right) - 1\right] \geq 0 \ . \tag{PLS}$$

We can now characterize the optimal policy schemes as follows:

[19] The term "old" refers to those entrepreneurs that would have chosen entrepreneurship rather than working in production even without the increase in basic research investments.

Proposition 3

Suppose the government maximizes aggregate consumption using (t_L, t_P, t_H, L_B) *as policy instruments. Then:*

(i) If and only if condition (PLS) *is satisfied, there will be an entrepreneurial economy with* $\tau^* = \bar{\tau}$, $L_B^* = \tilde{L}_B(\bar{\tau})$ *and* $L_E = 1 - \frac{1}{\bar{\tau} b \chi\left(\tilde{L}_B(\bar{\tau})\right)}$.

(ii) Else, there will be a stagnant economy with $\tau^* = \underline{\tau}$, $L_B^* = 0$ *and* $L_E = 0$.

We next analyze condition (PLS) more closely in order to deduce when an entrepreneurial economy is likely to prevail.

Corollary 1

Suppose the government maximizes aggregate consumption using (t_L, t_P, t_H, L_B) *as policy instruments. Then, the higher* m, b, *and* $\bar{\tau}$, *and the lower* γ, *the more likely it is that an entrepreneurial economy prevails.*

The proof of Corollary 1 is given in Appendix A.3. Corollary 1 implies that the more valuable innovations are, i.e. the higher is m and the lower is γ, the more likely it is that we will observe an entrepreneurial economy. Further, an entrepreneurial economy is the more likely the higher is the maximum admissible level of τ, $\bar{\tau}$, and the higher are the utility benefits (the lower are the utility cost) derived from becoming an entrepreneur, i.e. the higher is b.

5.3 Optimal policy without lump-sum taxes and transfers

In the previous section, separating the choice of L_B from that of the ratio between labor and profit taxes as captured in τ was feasible. We now ask whether this is always possible even when lump-sum taxes or transfers are not available. This means that given optimal values of L_B and τ we can always find values of t_L and t_P resulting in the desired value of τ and satisfying the budget constraint

$$wL_B = wt_L\left[\bar{L} - L_E\right] + t_P\left[\pi_y + \eta(L_B)L_E\pi_{xm}\right]. \tag{14}$$

Using the equilibrium values of π_y and π_{xm}, the budget constraint can be rewritten as

$$L_B = t_L\left[\bar{L} - L_E\right] + t_P\left[\frac{\alpha}{1-\alpha}L_y - m + L_E\chi(L_B)\right]. \tag{15}$$

The right-hand side of equation (15) corresponds to the tax revenue in working hour equivalents. It will subsequently be denoted by TR.

The definition of τ yields $t_L = 1 - \frac{1-t_P}{\tau}$. Inserting into equation (15) and solving for t_P, we obtain that the choice of L_B and τ can be separated only if this value of t_P, which we denote by $\tilde{t}_P$, is in the feasible range $[\underline{t}_P, \bar{t}_P]$ and $\tilde{t}_L = 1 - \frac{1-\tilde{t}_P}{\tau}$ is in $[\underline{t}_L, \bar{t}_L]$.[20] In the previous section we have seen that in the setting with lump-sum taxes either $\left(\bar{\tau}, \tilde{L}_B(\bar{\tau})\right)$ or $(\underline{\tau}, 0)$ is optimal. In this section we assume $\underline{t}_L = \underline{t}_P = 0$ as we like to allow the government to not provide basic research if desired. Then, by Assumption 1(ii), the policy choice $t_L = t_P = L_B = 0$ allows the realization of a stagnant economy also without lump-sum taxes.[21] By contrast, $\left(\bar{\tau}, \tilde{L}_B(\bar{\tau})\right)$ is not feasible in general in the setting without lump-sum taxes as it would require that $\left(\bar{t}_L, \underline{t}_P, \tilde{L}_B(\bar{\tau})\right)$ exactly satisfies equation (15).

As in the previous section, we will solve the government's maximization problem in two steps. First, we consider the optimal tax policy given that a certain level of basic research needs to be financed. Consider the set of all tax policies $\mathcal{T}(L_B)$ consisting of vectors (t_P, t_L), with $0 \leq t_P \leq \bar{t}_P$, $0 \leq t_L \leq \bar{t}_L$ and satisfying the budget constraint (15).[22] We focus on affordable basic research investments, i.e. $\mathcal{T}(L_B) \neq \emptyset$. For each such L_B, the policies in $\mathcal{T}(L_B)$ define a feasible range of τ. It will turn out that the upper bound $\bar{\tau}_O$ will be reached by using the labor income tax to finance basic research and levying a positive profit tax only if a ceteris paribus increase in t_L cannot be used to finance additional basic research. With the opposite prioritization of taxes, i.e. profit tax first and labor tax only if necessary, we will obtain the lower bound $\underline{\tau}_O$. In general, we refer to a *pecking order of taxation* if one tax is used (e.g. the labor income tax) and the other tax (e.g. profit tax) is levied only if an increase of the former cannot be used further to finance the public good. The latter case may occur if the prioritized tax reached its upper constitutional limit or is located at the decreasing part of the Laffer curve for TR. As indicated above, there are two pecking orders of taxation in our model: use labor income tax first and profit tax only if an increase in t_L cannot be

[20] The exact formula for $\tilde{t}_P$ is

$$\tilde{t}_P = \left(L_B + \frac{1-\tau}{\tau}(\bar{L} - L_E)\right) / \left(\frac{\alpha}{1-\alpha} L_y - m + L_E \chi(L_B) + \frac{\bar{L} - L_E}{\tau}\right).$$

[21] Cf. footnote 18

[22] Note that for $L_B > 0$, a necessary condition for the government budget constraint to be satisfied is that the policy choice satisfies the positive profit condition (PPC).

used to finance additional basic research and vice versa.

By definition, all policies in $\mathcal{T}(L_B)$ satisfy the government's budget constraint. However, depending on the implied level of τ, the tax policies entail different levels of entrepreneurship and consequently different output levels. Entrepreneurship increases aggregate consumption if $\chi(L_B) \geq 1$, i.e., if $L_B \geq L_{B,min}$. In this case, the government's tax policy aims at maximizing entrepreneurship by maximizing τ with the pecking order using labor income tax first. By contrast, the opposite pecking order will be applied to minimize entrepreneurship if $\chi(L_B) < 1$.[23] We formalize these insights in Proposition 4.

Proposition 4 (Pecking Order of Taxation)
Consider a government that maximizes aggregate consumption and finances a given amount of basic research L_B using (t_L, t_P) as tax measures. Suppose $\mathcal{T}(L_B) \neq \emptyset$. Then:

(i) If $L_B \geq L_{B,min}$, basic research should be financed using a pecking order with labor income tax first. In particular, $t_P > 0$ only if TR cannot be increased further by a ceteris paribus increase of t_L.

(ii) If $L_B < L_{B,min}$, basic research should be financed using a pecking order with profit tax first. In particular, $t_L > 0$ only if TR cannot be increased further by a ceteris paribus increase of t_P.

A proof of Proposition 4 is given in appendix A.4.

Proposition 4 characterizes the optimal tax policies to finance a given amount of basic research L_B. We will now use the optimal tax policies to determine the optimal provision of basic research. For this purpose, it is again convenient to consider first the constrained problem for $L_B \geq L_{B,min}$. Then the government's tax policy maximizes τ for each given L_B. Inserting $\bar{\tau}_O(L_B)$ into its objective, the government's constrained problem boils down to

$$\max_{\{L_B \geq L_{B,min}\}} \quad L_y(L_B, \bar{\tau}_O(L_B)) \ .$$

[23] In principle, there could be several tax schemes that fully deter entrepreneurship. If the government is indifferent between such tax policies, we will assume that it chooses $\underline{\tau}_O$.

Then we obtain as a necessary condition for a maximum

$$\frac{\partial L_y(L_B, \bar{\tau}_O(L_B))}{\partial L_B} + \frac{\partial L_y}{\partial L_E}\frac{\partial L_E}{\partial \tau}\frac{\partial \bar{\tau}_O(L_B)}{\partial L_B} \leq 0 \ , \tag{16a}$$

$$\left(\frac{\partial L_y(L_B, \bar{\tau}_O(L_B))}{\partial L_B} + \frac{\partial L_y}{\partial L_E}\frac{\partial L_E}{\partial \tau}\frac{\partial \bar{\tau}_O(L_B)}{\partial L_B}\right)(L_B - L_{B,min}) = 0. \tag{16b}$$

The first partial derivative of the objective L_y with respect to L_B corresponds to the necessary condition for maximization of aggregate consumption when lump sum taxes and transfers are feasible (13a). The second summand captures the effect of L_B on τ implying that a marginal increase of basic research additionally influences the amount of entrepreneurs making use of it via the tax scheme. The sign of $\frac{\partial L_y}{\partial L_E}$ is positive for $L_B > L_{B,min}$. For $L_E > 0$, the term $\frac{\partial L_E}{\partial \tau}$ is clearly positive as indicated by the equilibrium value of L_E given in (5). Finally, the last expression represents the marginal effect of basic research on $\bar{\tau}_O$ as implied by the government budget constraint. The sign of this effect depends on two interdependent factors: First, it depends on whether or not an increase in L_B requires additional funding. An increase in L_B might in principle generate additional tax returns in working hour equivalents that exceed the increase in L_B. Second, it depends on how exactly basic research is financed: via a change in labor income or via a change in profit taxes. Suppose for example that both tax measures are located at the increasing part of the Laffer curve and that an increase in basic research requires additional funding. Then, with the pecking order $\bar{\tau}_O$, the government uses the labor tax to finance additional basic research implying $\frac{\partial \bar{\tau}_O(L_B)}{\partial L_B} = \frac{\partial \tau}{\partial t_L}\frac{\partial t_L}{\partial L_B} > 0$. If by contrast an increase in L_B cannot be funded via an increase in the labor tax either because it reached its upper bound or because it is located at the decreasing part of the Laffer curve, then additional basic research will be financed by an increase in the profit taxes and/or a decrease in the labor tax and the last expression becomes negative.

Let us denote the solution of the government's problem constrained to $L_B \geq L_{B,min}$, i.e. given the pecking order with labor taxes first, by $L_{B,\bar{\tau}_O}$. Again, note that $L_{B,\bar{\tau}_O} > L_{B,min}$ implies that (16a) holds with equality.

Now we consider the government's problem restricted to $L_B < L_{B,min}$ implying a pecking order with profit taxes first, $\underline{\tau}_O$. Since in this case entrepreneurship affects consumption negatively, the government will prevent inefficient entrepreneurship by providing no basic research.[24] Hence, the solution to this restricted optimization problem will be $(L_{B,\underline{\tau}_O} = 0, \underline{\tau}_O = 1)$.

[24]Note that the government is able to deter inefficient entrepreneurship by not providing any basic research according to Assumption 1. Via the budget constraint $L_B = 0$ implies $t_P = t_L = 0$.

Consequently, the government will implement $L_{B,\bar{\tau}_O} > L_{B,min}$ if and only if basic research increases the entrepreneurs' innovation probability enough to compensate for the investments in basic research and the labor "lost" to entrepreneurship. That is, if and only $L_{B,\bar{\tau}_O}$ satisfies:

$$-L_{B,\bar{\tau}_O} + \left[1 - \frac{1}{\bar{\tau}_O b \; \chi\left(L_{B,\bar{\tau}_O}\right)}\right] \left[\chi\left(L_{B,\bar{\tau}_O}\right) - 1\right] \geq 0 \; . \tag{PLS2}$$

Otherwise the government will implement policy $L_B = t_P = t_L = 0$. Proposition 5 summarizes our results.

Proposition 5

Suppose the government maximizes aggregate consumption using (t_L, t_P, L_B) as policy instruments. Then:

(i) If and only if condition (PLS2) is satisfied, there will be an entrepreneurial economy with $\tau^* = \bar{\tau}_O$, $L_B^* = L_{B,\bar{\tau}_O}$ and $L_E = 1 - \frac{1}{\bar{\tau}_O b \; \chi\left(L_{B,\bar{\tau}_O}\right)}$.

(ii) Else, there will be a stagnant economy with $t_L^* = t_P^* = 0$, $L_B^* = 0$ and $L_E = 0$.

6 The political economy of financing basic research

So far, we have taken the viewpoint of a government seeking to maximize aggregate consumption, without caring about distributional effects. Our analyses of the previous sections suggested that innovation stimulating investments in basic research should be complemented by a pecking order of taxation. Obviously, such innovation policies might have substantial distributional effects. In this section we explore these distributional effects and whether policies fostering entrepreneurship are politically viable.

In our model, the government has two main policy areas at its discretion to foster entrepreneurship and innovation in the economy: basic research and tax policy. These policies have direct distributional effects: (a) labor income and profit taxes allow for redistribution of wealth between workers on the one hand and entrepreneurs and shareholders of the final good producer on the other hand. (b) Basic research investments have a direct effect on entrepreneurs by improving their chances of success. However, these direct effects are accompanied by important general equilibrium feedback effects. In particular, basic research investments support labor-saving technological progress in the intermediate good sectors. As a consequence of innovations, labor is set free in the intermediate good sectors and additionally supplied to final good production. This increases output and the profits of the representative final good producer but lowers wages.[25,26] Hence, while ownership in the final good firm is irrelevant for the consumption maximizing policies, it is crucial for the distributional effects of such policies.

6.1 A median voter framework

We take the political economy perspective of a median voter model and ask whether the median voter will support an entrepreneurial economy. More precisely, we order the population according to shareholdings and assume that the median voter is a worker with $\hat{s}$ shares of the final good producer and ask whether the median voter prefers investments in basic research and entrepreneurship relative to a stagnant economy. Of course, if the median voter possesses a sufficient amount of shares of the final good firm, she will support an entrepreneurial economy maximizing aggregate consumption. The more realistic and interesting case is when income is skewed such that the median voter possesses less than the per capita claims on final good profits.[27] This includes

[25]These implications are consistent with the common trend across industrialized economies that labor income - in particular labor income of low skilled workers - as a share of total value added is decreasing over time. Timmer et al. (2010), for example, show that for the European Union worker's share in total value added decreased from 72.1% in 1980 to 66.2% in 2005. In the US, this share decreased from 66.8% to 63.2%. At the same time, the share of high-skilled workers' income in total value added increases rapidly over time: In the EU, this share increased from 8.3% in 1980 to 16.0% in 2005, whereas in the US it increased from 18.5% to 30.4%.

[26]With divisible intermediate goods, labor saving technological progress in the intermediate good sector would not result in a decrease in wages. Still, there would be a conflict between efficiency and equality in our economy as discussed here, at least if innovations are non-drastic: With divisible intermediates, an innovating entrepreneur would preferably charge a price $p_i = \frac{mw\gamma}{\alpha}$. For $\gamma > \alpha$ this is not feasible due to competition from the standard technology and the innovating entrepreneur sets price $p_i = mw$ instead. In that sense innovations are *non-drastic*. $p_i = mw \; \forall i$, implies that $w = [1-\alpha]^{(1-\alpha)} \left[\frac{\alpha}{m}\right]^{\alpha}$ and hence the wage rate is independent of the innovation step γ in the economy. Intuitively, wages depend on the marginal product of labor in final good production and hence on the ratio of labor to intermediates. With constant intermediate good prices, this is the same irrespective of the production technology in the intermediate good sector. The monopoly distortion in the intermediate good sector prevents the introduction of more intermediate good-intense production processes in final good production and hence a higher marginal product of labor. Note that with constant gross wages, a conflict between equality and efficiency follows from tax policies: in the entrepreneurial economy, workers contribute to the provision of basic research and hence end-up with lower net wages than in the stagnant economy where government spendings are zero. Obviously, with constant returns to scale and divisible intermediates, the final good producer earns zero profits and benefits from innovation accrue with the successful entrepreneurs. Hence, shareholdings in the final good firms do not matter for the distribution of the gains from innovation.

[27]In connection with Proposition 6 below we rationalize the application of the median voter model for the case of all potential entrepreneurs having at least as many shareholdings as the median voter. Alternatively, we may think of a society where half (plus ε) of the population is identical: workers with a certain amount of stocks of the final good firm.

the special case where the median voter is a worker without any stocks.[28] The resulting trade-off is obvious: In the stagnant economy, wages are higher and the median voter can maximally redistribute profits without considering incentives for occupational choice by potential entrepreneurs. On the other hand, the tax base is higher in an aggregate consumption beneficial entrepreneurial economy, potentially allowing for higher redistributional transfers even if profit tax rates are lower. Hence, an entrepreneurial economy might be preferred to a stagnant economy with maximal profit tax.

In this section, we return to our base-case allowing for lump-sum taxes.[29] To simplify the exposition, we furthermore assume common tax bounds for labor income and for profit taxes. That is, we assume $\bar{t}_P = \bar{t}_L = \bar{t} \in [0, 1-\varepsilon]$ and $\underline{t}_P = \underline{t}_L = \underline{t} \in [0, 1-\varepsilon]$ for some arbitrarily small $\varepsilon > 0$ and $\bar{t} \geq \underline{t}$. Consequently, $\tau \in [\underline{\tau}, \bar{\tau}] \equiv \left[\frac{1-\bar{t}}{1-\underline{t}}, \frac{1-\underline{t}}{1-\bar{t}}\right]$ and $\bar{\tau} < \infty$. Further we use $\hat{s} \in \left[0, \frac{1}{\bar{L}}\right)$ to denote the share of the profits of the final good firm that the median voter can claim.[30] Consequently, the median voter's income is given by

$$\begin{aligned} I &= (1-t_L)w + (1-t_P)\hat{s}\pi_y + t_H \\ &= w + \hat{s}\pi_y + NT \ , \end{aligned} \tag{17}$$

where $w + \hat{s}\pi_y$ is the median voter's gross income and $NT = t_H - t_L w - t_P \hat{s}\pi_y$ denote net transfers to him. We obtain the lump sum transfers, t_H, from the government's budget constraint as

$$t_H = \frac{1}{\bar{L}} \left[t_L w \left(\bar{L} - L_E \right) + t_P \left(\pi_y + \eta(L_B) L_E \pi_{xm} \right) - w L_B \right] \ . \tag{18}$$

Substituting the profits by their equilibrium values as provided in Proposition 1 and

[28] For example, a fraction $\frac{1}{2} < \mu < 1$ of the population are workers who do not own shares in the final good producer. The situation with a majority of the population being workers who are not engaged in the stock market is in line with empirical evidence on stock market participation rates. For example, Guiso et al. (2008) show for a selection of 12 OECD member states percentages of households that are engaged in the stock market. Even if indirect stockholdings are also considered, Sweden is the only country where a majority of households is engaged in the stock market with most countries having a share of households that is engaged in the stock market of less than one third.

[29] Without lump-sum taxes, a redistribution via tax policies is no longer feasible and it turns out that an aggregate output stimulating entrepreneurial economy is no longer supported by the median voter if shareholdings are sufficiently skewed. In particular, the median voter will always prefer the stagnant economy over the entrepreneurial economy if he owns less than a fraction $\frac{\bar{L}}{\bar{L}+(1-\gamma)m}$ of the per-capita shares in the final good producer. Intuitively, in the proof of Proposition 8 we argue that in this case the gross income of the median voter is decreasing in aggregate output and hence he can be no better off in the entrepreneurial economy than in the stagnant economy with $t_L = t_P = 0$. We note that the condition discussed here is sufficient but never necessary for our result.

[30] With the population being ordered according to shareholdings in the final good producer we must have $\hat{s} \in \left[0, \frac{2}{\bar{L}}\right]$. We assume $\hat{s} \in \left[0, \frac{1}{\bar{L}}\right)$ as discussed above.

using per capita labor shares denoted by lower case l, e.g. $l_Y = L_Y/\bar{L}$, we obtain for the net transfers to the median voter

$$NT = w\left[t_P\left[\left(\frac{\alpha}{1-\alpha}l_Y - l_m\right)(1-s) + \chi(L_B)l_E\right] - t_L l_E - l_B\right] , \tag{19}$$

where $s = \hat{s} * \bar{L}$, and $l_m = m/\bar{L}$ denotes the share of labor employed in intermediate good production in the stagnant economy. Note that $s < 1$ given our assumption that the stocks of the final good firm are concentrated in the hands of a minority. Using the same notation as in (19), we can write the median voter's gross income as $w[1 + s(\frac{\alpha}{1-\alpha}l_Y - l_m)]$. An important observation is that for given basic research investments, the level of entrepreneurship and production is determined only by the ratio $\tau = \frac{1-t_P}{1-t_L}$ but not by the absolute values of tax rates. Hence, the median voter's gross income is also uniquely determined by the choices of τ and L_B. The levels of the labor and profit tax rates only matter for the degree of redistribution as apparent in (19).

Consequently, we can determine the median voter's most preferred policy by the following procedure: First, we derive the optimal amount of redistribution by choosing the levels of t_L and t_P for given τ and L_B. This allows us to write the median voter's objective as a function of τ and L_B and consequently to determine the median voter's most preferred levels of τ and basic research investments L_B.

With τ given, we can substitute t_L by $1 - (1-t_P)/\tau$ in expression (19) reflecting the net transfers. Then, taking the derivative of the net transfers with respect to t_P yields:

$$DNT \equiv \left.\frac{\partial NT}{\partial t_P}\right|_{\tau} = w\left[(\frac{\alpha}{1-\alpha}l_Y - l_m)(1-s) + \chi(L_B)l_E - \frac{l_E}{\tau}\right] . \tag{20}$$

Note that with lump-sum transfers, a marginal increase in the profit tax constitutes a redistribution of profits (from entrepreneurs and the final good firm) to workers while an increase in the labor tax redistributes from workers to entrepreneurs.[31] The redistribution of profits is captured by the first two summands in (20), where the first summand reflects the additional redistribution of the final good firm's profits, and the second summand represents the additional redistribution of entrepreneurial profits. By the assumption that the median voter is a worker, redistribution of entrepreneurial profits is beneficial for him. The factor $1-s$ indicates that the redistribution of the

[31]The increase in the labor tax does not per se describe a redistribution towards the owners of the shares of the final good firm, as these are also either workers or entrepreneurs.

final good firm's profits is only favorable if the share of profits he can claim is less than $1/\bar{L}$. The latter results from the fact that transfers are lump sum. Finally, keeping τ constant, an increase in the profit tax t_P by a marginal unit must be matched by an increase in the labor tax t_L of $1/\tau$. The resulting amount of redistribution of labor income to entrepreneurs is captured by the last summand in DNT.

If DNT is positive, net transfers for the median voter are maximized by the highest possible profit tax rate, while the opposite is true if DNT is negative. However, the optimal choice of t_P (and t_L) in consequence will depend on the particular value of τ. The following table shows the optimal levels of t_P and t_L depending on DNT and τ. Note that since profits of the final good firm are non-negative ($w\left(\frac{\alpha}{1-\alpha}l_Y - l_m\right) \geq 0$), the case where $DNT < 0$ and $\tau \geq 1$ can only occur if entrepreneurship is inefficient (i.e. $\chi(L_B) < 1$) and/or $s > 1$.

	$\tau \geq 1$	$\tau < 1$
$DNT \geq 0$	$t_L = \bar{t}$, $t_P = 1 - \tau(1-\bar{t})$	$t_L = 1-(1-\bar{t})/\tau$, $t_P = \bar{t}$
$DNT < 0$	$t_L = 1-(1-\underline{t})/\tau$, $t_P = \underline{t}$	$t_L = \underline{t}$, $t_P = 1-\tau(1-\underline{t})$

Table 1: Optimal labor and profit tax rates given τ and L_B.

We use $\hat{t}_L(\tau, L_B)$ and $\hat{t}_P(\tau, L_B)$ to refer to the optimal labor and profit tax rates for given τ and L_B. Using these tax rates, we can write the net transfers and consequently the median voter's income as a continuous function of τ and L_B.

Lemma 2

Using $\hat{t}_L(\tau, L_B)$ and $\hat{t}_P(\tau, L_B)$, the median voter's income is a continuous function on $[\underline{\tau}, \overline{\tau}] \times [0, \bar{L}]$.

The proof is given in appendix A.5. Note that the median voter's income is not differentiable at the values of τ and L_B where $DNT = 0$. With these results, we will now move on to the second part of the median voter's maximization problem concerning the level of τ and the amount of basic research investments. Using Lemma 2, the median voter seeks the maximum of a continuous function over a compact set. Hence, by the Weierstrass extreme value theorem, the maximum will be reached. However, the set of maximizers may not be single-valued. It is instructive to discuss some properties of the median voter's income maximization problem, by approaching it in the two-step procedure used in the previous sections.

Consider the optimization of the median voter's income (17) with respect to τ for given basic research investments L_B:

$$\max_{\tau} \quad I(\tau, L_B) = w(\tau, L_B)\left[1 + s\left(\frac{\alpha}{1-\alpha} l_Y(\tau, L_B) - l_m\right)\right] + NT(\tau, L_B) \qquad (21)$$

Regarding a marginal increase in τ, the median voter's income is affected as follows[32]

$$\frac{dI(\tau, L_B)}{d\tau} = \frac{\partial NT}{\partial \hat{t}_P}\frac{\partial \hat{t}_P(\tau, L_B)}{\partial \tau} + \frac{\partial NT}{\partial \hat{t}_L}\frac{\partial \hat{t}_L(\tau, L_B)}{\partial \tau} + \frac{\partial I(\tau, L_B)}{\partial l_E}\frac{\partial l_E}{\partial \tau}\ . \qquad (22)$$

Note that $\frac{dI(\tau,L_B)}{d\tau}$ must be zero for an interior solution τ other than the critical value of τ implying $DNT = 0$. An increase in τ has two fundamental effects: it increases the relation between labor and profit taxes and it (weakly) increases the number of entrepreneurs. The first two summands in (22) reflect the decline in redistribution from profits to labor income due to the relatively lower profit taxes. Note that one of the summands is zero as either $\hat{t}_P$ or $\hat{t}_L$ remains at the boundary of the feasible set $[\underline{t}, \bar{t}]$. The last term in (22) captures the effect of an increase in the number of entrepreneurs on the median voter's income.[33] In the case where entrepreneurship is efficient, i.e. $\chi(L_B) > 1$, an increase in entrepreneurship will increase profits and total output but will lead to a lower wage rate. Consequently, a median voter with a small amount of stocks faces the following trade-off regarding τ. On the one hand, a marginally higher level of τ decreases her gross income (as the wage payments are the major income source) and lowers the share of profits that are redistributed.[34] On the other hand, a larger τ increases total output and therefore the tax base for the profit tax. This reflects a standard Laffer-curve trade-off.

As the set of maximizers may contain several values of τ, we cannot proceed as in the previous sections by defining a function $\tau(L_B)$, inserting back into the objective function and then solving for the optimal value of L_B. Instead, we could derive the correspondence $L_B(\tau)$ which maximizes the median voter's income with respect to basic research investments for a given level of τ. Candidates of optimal policies for the median voter will lie in the intersection of the two correspondences. Those with the

[32]Note that the terms $\frac{\partial \hat{t}_P(\tau,L_B)}{\partial \tau}$ and $\frac{\partial \hat{t}_L(\tau,L_B)}{\partial \tau}$ differ according to the different cases in Table 6.1. At the critical values τ_c, as defined in the proof of Lemma 2, and $\tau = 1$, equation (22) refers to the right-sided derivative.

[33]Note that for small values of L_B and τ, L_E will remain at zero in response to a marginal increase in τ.

[34]Obviously, if τ is increased via an increase of t_L rather than a decrease of t_P, a higher share of labor income is redistributed to entrepreneurs.

highest income level then constitute the median voter's preferred policies. As in the previous section, we refer to an *entrepreneurial economy* if $L_B > L_{B,min}$ and $L_E > 0$ with total output y exceeding the total output when $L_E = L_B = 0$. We speak of a *stagnant economy* if $L_E = L_B = 0$. With inefficient entrepreneurship, i.e. $\chi(L_B) < 1$, an economy's total output will be less than the output without basic research and entrepreneurship. As inefficient entrepreneurship decreases the labor input in final good production and hence increases wages, a median voter with little or no stocks may find it beneficial to foster such inefficient entrepreneurship by investing in basic research. Even without entrepreneurship, a median voter may try to maximize wages by investments in basic research to reduce the labor supply in final good production. As this scenario might not be the most realistic one, we neglect it in the following and concentrate on stagnant economies with $L_E = L_B = 0$.[35,36]

We will now examine the question under which conditions the median voter will support an entrepreneurial economy. This is the case if and only if his income in the entrepreneurial economy

$$I^E = w\left[1 + (\frac{\alpha}{1-\alpha}l_Y - l_m)(s + t_P(1-s)) + l_E(t_P\chi(L_B) - t_L) - l_B\right] \tag{23}$$

is at least as high as his income in the stagnant economy

$$I^S = w^S\left[1 + \left(\frac{\alpha}{1-\alpha}l_Y^S - l_m\right)(s + t_P(1-s))\right] . \tag{24}$$

We start our analysis with the following observation.

Proposition 6

Suppose a worker with shareholdings $s^\star$ prefers an aggregate consumption beneficial entrepreneurial economy over the stagnant economy. Then, so do all voters with shareholdings $s \geq s^\star$.

A formal proof of Proposition 6 is given in Appendix A.6. Intuitively, the higher a worker's shareholdings, the more he can benefit from the increase in final good producer's profits associated with an efficient entrepreneurial economy. The result extends to potential entrepreneurs with shareholdings $s \geq s^\star$ as they are all worker in the stagnant economy. Then, if they remain workers in the entrepreneurial economy,

[35] Note that with maximal redistribution in the stagnant economy, $t_P = \bar{t}$ and $t_L = \underline{t} \leq \bar{t}$ implies $\tau \leq 1$ and, hence, $L_B = 0$ implies $L_E = 0$ by Assumption 1.

[36] We may think of an agenda setter seeking to implement growth enhancing policies and who needs the support of the median voter.

their trade-off is just the same as the one faced by a worker with same shareholdings. If by contrast they opt to become entrepreneurs, then they must prefer this option over being worker and the result follows. Note that Proposition 6 rationalizes the application of the median voter model here if all potential entrepreneurs have at least as many shareholdings in the final good producer as the median voter.[37]

With regard to the political support for an entrepreneurial economy, Proposition 6 loosely speaking implies that this is the higher the more equally distributed shareholdings are.[38] In the following proposition we take an alternative view and show that the median voter will always prefer the entrepreneurial economy over the stagnant if the constitutional upper bounds on taxation are sufficiently close to 1, irrespective of his shareholdings $s < 1$.

Proposition 7
If there exists an entrepreneurial economy $(\hat{\tau}, \hat{L}_B)$ with higher aggregate output than a stagnant economy, then there also exists a constitutional upper limit of tax rates $\bar{t}$ such that $\hat{\tau} \in [\underline{\tau}, \overline{\tau}]$ and the median voter will prefer the entrepreneurial policy over a stagnant economy.

A formal proof is given in Appendix A.7. The intuition is straightforward: with $\bar{t}$ sufficiently close to 1, it is feasible to implement any τ with t_P close to 1. Hence, all profits can effectively be redistributed in the entrepreneurial economy via the lump-sum tax, allowing all workers to benefit from the increase in aggregate output.

The main insight of Proposition 7 is that incentives for entrepreneurship by a high value of τ as well as redistribution of profits by a sufficiently high value of t_P can be reconciled, if the constitutional upper boundary on tax rates is very close to 1. However, if the upper and lower bounds on taxation are too low, providing both incentives for economic feasibility and redistribution for political viability of an entrepreneurial economy will not be possible.

Proposition 8
Let $\underline{t} = 0$. If $\bar{t}$ is sufficiently low, a median voter with $s \leq \frac{\bar{L}}{\bar{L}+(1-\gamma)m}$ will support a stagnant economy.

[37]With some entrepreneurs having less shareholdings than the median voter, the median voter model might no longer apply as in any entrepreneurial economy all entrepreneurs but the marginal are strictly better off as their working counterpart with same shareholdings. Hence, an entrepreneurial economy that is inferior to the stagnant for the median voter might be supported by some potential entrepreneurs with less shares.

[38]Note that the median voter is assumed to own less than the per capita shares.

A proof of Proposition 8 is given in Appendix A.8. Intuitively, the condition $s \leq \frac{\bar{L}}{\bar{L}+(1-\gamma)m}$ guarantees that labor income is the decisive component of the median voter's gross income. For sufficiently restrictive tax bounds, redistribution of profits via the lump-sum taxes can no longer compensate for the decrease in labor income associated with the entrepreneurial economy and the median voter prefers the stagnant economy.

Let $\underline{t} = 0$ and $s \leq \frac{\bar{L}}{\bar{L}+(1-\gamma)m}$ and fix any entrepreneurial policy $(\hat{\tau}, \hat{L}_B)$ with $\hat{L}_y \geq L_y^S$. Proposition 7 implies that this entrepreneurial economy is preferred over the stagnant economy by the median voter if $\bar{t}$ is sufficiently high. Proposition 8 implies that this is no longer the case if $\bar{t}$ is sufficiently low. In principle, there are two possibilities why this might happen: First, $\bar{t}$ might prevent sufficiently large transfers to the median voter. Second, for $\bar{t}$ too low $\hat{\tau}$ might no longer be available, i.e. we might have $\hat{\tau} \notin [\underline{\tau}, \bar{\tau}]$. Let us say that the entrepreneurial economy $(\hat{\tau}, \hat{L}_B)$ is feasible in the median voter framework if $\hat{\tau} \in [\underline{\tau}, \bar{\tau}]$ and if it is preferred over the stagnant economy by the median voter. Then, for every such entrepreneurial economy there must exist a threshold value $\bar{t}_c^l$ such that the entrepreneurial economy is no longer feasible if $\bar{t} < \bar{t}_c^l$ and a threshold value t_c^u such that the entrepreneurial economy is feasible if $\bar{t} \geq t_c^u$. We summarize these insights in the following Proposition and show that these two threshold values coincide.[39]

Proposition 9

Let $\underline{t} = 0$. For any entrepreneurial policy $(\hat{\tau}, \hat{L}_B)$ with $\hat{L}_y > L_y^S$, there exists a critical value $0 < \bar{t}_c < 1$ such that $\hat{\tau} \in [\underline{\tau}, \bar{\tau}]$ and the median voter with $s \leq \frac{\bar{L}}{\bar{L}+(1-\gamma)m}$ will prefer the entrepreneurial policy over the stagnant economy if and only if $\bar{t} \geq \bar{t}_c$.

The proof of Proposition 9 is given in Appendix A.9. Depending on which of the four quadrants of table 6.1 we are in, t_c may originate from either of the two potential reasons identified above. For entrepreneurial economies $\hat{\tau}$, $\hat{L}_B$ with $\hat{\tau} \geq 1$ and $DNT \geq 0$ the median voter's preference gives rise to the threshold value $\bar{t}_c$. To see this, note that with $\underline{t} = 0$, $\hat{\tau} \geq 1$ is possible as long as $\bar{t} \geq 1 - \frac{1}{\hat{\tau}}$. Moreover, for $\bar{t} = 1 - \frac{1}{\hat{\tau}}$ we have $t_P = 0$, implying that $NT < 0$ and hence that the median voter prefers the stagnant economy. It follows that the median voter prefers the entrepreneurial economy only if $\hat{\tau} < \bar{\tau}$, i.e. only if $\hat{\tau} \in [\underline{\tau}, \bar{\tau}]$. By contrast, $\hat{\tau} = \underline{\tau}$ defines $\bar{t}_c$ for entrepreneurial economies $\hat{\tau}$, $\hat{L}_B$ with $\hat{\tau} < 1$ and $DNT < 0$. In the proof of Proposition 9 we argue that the median voter always prefers this entrepreneurial economy over the stagnant

[39]In principle, $I^E - I^S$ may be non-monotonous and hence we might have $\bar{t}_c^l \neq \bar{t}_c^u$.

economy and hence $\bar{t}_c = 1 - \hat{\tau} > 0$. For the case of $\hat{\tau} < 1$ and $DNT > 1$ either of the previously discussed reasons might give rise to the threshold value $\bar{t}_c$.

Consider the set of feasible entrepreneurial policies (τ, L_B) satisfying $L_y > L_y^S$. According to Proposition 9, there exists a corresponding set of critical constitutional upper bounds $\bar{t}_c$. Denote the infimum of this set of critical tax rates by $\bar{t}_{inf}$. Then, the following Corollary follows immediately.[40]

Corollary 2
The median voter will opt for an entrepreneurial economy satisfying $L_y > L_y^S$ if and only if $\bar{t} \geq \bar{t}_{inf}$. Else, the median voter supports the stagnant economy.

Note that from Proposition 8 it follows that $\bar{t}_{inf} > 0$.

6.2 Discussion

In this section we have analyzed the political economy of financing basic research investments. We used a median voter framework and assumed that the median voter possesses an under-proportional share of stocks of the final good firm. Consequently, the median voter's income depends decisively on labor income. As the wage rate will be lower in an efficient entrepreneurial economy with labor saving innovation than in a stagnant economy, the former will not be supported by the median voter with sufficiently small shareholdings without any redistribution of profits (Proposition 8). This opens up a conflict between economic incentives for becoming an entrepreneur, which requires a low relation of tax rates on profits and labor income, and political viability which calls for a high profit tax rate. As we show in Proposition 7, both motives can be reconciled if the constitutional upper limit on tax rates is sufficiently large. The intuitive reason is that the economic incentive to become an entrepreneur depend on the relation between profit and labor taxes, while the redistribution necessary for political viability depends on the absolute levels of the profit and labor taxes. For our analysis we made the assumption that it is always possible to not levy any taxes, i.e. the constitutional lower limit of the tax rates is zero. Then there exists a threshold value for the constitutional upper limit of tax rates where entrepreneurial policies are politically precluded if the constitutional maximum tax rate is lower. Typically, low constitutional bounds on taxation are aimed at protecting the shareholders of firms from excessive

[40] Remember that we disregard policies with inefficient entrepreneurship and / or basic research.

expropriation. In our framework where labor-saving innovations take center stage, such low bounds on taxation may well prevent an entrepreneurial economy.

In our analysis, we have assumed that profit taxes are the same in the intermediate and in the final good sector. Of course, within our model, if distributional reasons prevent the existence of an entrepreneurial economy, it might be optimal to tax profits in the final good sector differently from those in the intermediate good sector. Such tax discrimination might be interpreted as different tax treatment of corporate versus non-corporate income found in the US, for example. It would allow compensating workers with the tax revenues from the beneficiaries of the final good producer's profits. Clearly, such compensations would also affect the occupational choice by potential entrepreneurs and hence they would have a deeper impact on optimal policy choices. For any choice of L_B such that $\chi(L_B) > 1$, it would be optimal to tax profits of the final good producer at the highest possible rate, make the median voter just indifferent between the status quo and the new policy choice (in order for the policy choice to be politically viable) and use the remainder of the output as incentives to entrepreneurs.

If tax differentiation between entrepreneurs and final good producers is not possible due to asymmetric information, for example, then progressive taxes might also be used to support the implementation of an entrepreneurial economy. Of course, this would only be a viable option if profits of the representative final good producer exceeded those of the successful entrepreneur. From Proposition 1 we conclude that this is the case if and only if

$$L_y > (2-\gamma)\frac{1-\alpha}{\alpha}m \ .$$

7 Maximization of Aggregate Welfare

In this section we analyze the case of a government that aims to maximize aggregate utility rather than aggregate consumption. We reintroduce lump-sum taxes, allowing again the government to separate the choice of the optimal amount of basic research from the optimal financing scheme. In our model, aggregate utility, W, is given by

$$W = (1-t_P)\pi_y + \int_0^{L_E} (1-t_P)\lambda_k \eta(L_B)\pi_{xm} - t_H \, dk + \int_{L_E}^{\bar{L}} (1-t_L)w - t_H \, dk \ . \qquad (25)$$

Combining (25) with the government budget constraint, (4), the labor market clearing condition, (7), and the aggregate income identity, $y = \pi_y + \eta(L_B)L_E\pi_{xm} + (L_x + L_y)w$, yields

$$W = y + (1 - t_P)\eta(L_B)\pi_{xm} \int_0^{L_E} \lambda_k - 1 \, dk \ .$$

Substituting y and π_{xm} by their respective equilibrium values given in part (i) of Proposition 1 and solving the integral using $\lambda_k = (1 - k)b$ it follows

$$W = L_y^{1-\alpha} + (1 - t_P)\eta(L_B)(1 - \gamma)m(1 - \alpha)bL_y^{-\alpha}L_E \left[1 - \frac{1}{b} - \frac{L_E}{2}\right] \ . \tag{26}$$

The government's decision problem is to maximize (26) subject to the non-negativity constraint of the final good producer and equilibrium conditions (1) and (3) given in Proposition 1.

Comparing the expression for aggregate welfare given in equation (26) with the expression for aggregate consumption given in equation (**??**) it becomes apparent that aggregate welfare corresponds to aggregate consumption plus the immaterial benefits (cost) of entrepreneurs. This immaterial utility term is scaled by $(1 - t_P)$, i.e. profit taxes allow the government to directly affect this term. So when maximizing aggregate welfare, not only the relative size of $(1 - t_P)$ compared to $(1 - t_L)$ matters, but also its absolute size. The imposition of labor income taxes affects the occupational choice of potential entrepreneurs and hence the equilibrium number of entrepreneurs that exploit the basic research provided. The imposition of profit taxes also influences the occupational choice of potential entrepreneurs, but in addition affects the utility received by those who opt to become entrepreneurs. Proposition 10 shows that this implies that in any welfare optimum with strictly positive entrepreneurship at least one tax measure is located at the boundary of its feasible set. The intuition is that for any strictly interior combination of tax measures, there is a continuum of combinations of t_L and t_P yielding the same τ and hence the same level of entrepreneurship in the economy. Now, if for a given τ the immaterial utility term in the aggregate welfare is positive, then the welfare maximizing combination of t_L and t_P yielding this τ is the t_P-minimizing which requires that either $t_L = \underline{t}_L$ or $t_P = \underline{t}_P$ or both. A similar argument reveals that either $t_L = \bar{t}_L$ or $t_P = \bar{t}_P$ or both if the immaterial utility term in the aggregate welfare is negative. The case where the aggregate immaterial utility term is exactly equal to zero is somewhat more involved. The intuition here is that in this case aggregate welfare reduces to aggregate consumption which we have shown previously to be maximized at either $\bar{\tau}$ or $\underline{\tau}$.

Proposition 10

Let (t_L^*, t_P^*, L_B^*) be a welfare optimum such that $\tau^* := \frac{1-t_P^*}{1-t_L^*} > \frac{1}{\chi(L_B^*)b}$. Then at least one tax measure is at the boundary of its feasible set, i.e. $t_P^* = \underline{t}_P$, $t_P^* = \overline{t}_P$, $t_L^* = \underline{t}_L$ or $t_L^* = \overline{t}_L$.

Proposition 10 follows immediately from Proposition 13 in appendix B. It implies that no interior optimum exists for tax policies. We next characterize the optimal tax policy for a given L_B in more detail. Consider the expected marginal reduction of labor used in intermediate good production from marginally increasing the measure of entrepreneurs: $\chi(L_B)$. $\chi(L_B) \gtreqless 1$ has three important implications for the welfare optimal policy: First, $\chi(L_B) \gtreqless 1$ determines whether increasing the number of entrepreneurs, L_E, increases or decreases in expectation the labor available for final good production, L_y, and hence output of the final good. From this it follows that $\chi(L_B) \gtreqless 1$ determines whether or not increasing the number of entrepreneurs rises the monopoly profits of successful entrepreneurs and hence escalates the immaterial utility from being entrepreneur. In particular, if $\chi(L_B) > 1$, then monopoly profits decrease with entrepreneurship in the economy which dampens the immaterial utility of each entrepreneur and hence aggregate immaterial utility in the economy. Finally, for $b \geq 1$, $\chi(L_B) \gtreqless 1$ determines whether given tax neutrality, i.e. $t_L = t_P$, the marginal entrepreneur earns positive or negative immaterial utility from being entrepreneur.

As we have argued previously, depending on whether or not the immaterial utility term in the aggregate welfare is positive, it is optimal to either implement the desired τ in the t_P-minimizing or the t_P-maximizing way. We now take on the opposite viewpoint and consider the optimal level of τ given t_P and show that tax neutrality, i.e. a tax policy satisfying $t_L = t_P$, is not welfare maximizing in general.

For t_P given, τ is determined by t_L which only affects entrepreneurship in the economy. In particular, the following relationship between the marginal effect of labor income taxes and entrepreneurship on aggregate welfare holds:

$$\frac{\partial W}{\partial t_L} = \begin{cases} \frac{\partial W}{\partial L_E} \frac{1}{(1-t_P)\chi(L_B)b} & , \quad \text{if } \frac{1-t_L}{(1-t_P)\chi(L_B)b} \leq 1 \\ 0 & , \quad \text{if } \frac{1-t_L}{(1-t_P)\chi(L_B)b} > 1 \; . \end{cases}$$

We will make use of this close relationship between τ, t_L, and L_E for t_P and L_B given and analyze welfare effects of entrepreneurship directly which yields the most insights.

The partial derivative of W with respect to L_E is given by:

$$\frac{\partial W}{\partial L_E} = (1-\alpha)L_y^{-\alpha}\Bigg\{(\chi(L_B)-1)+(1-t_P)\chi(L_B)b \\ \left[\left(1-\frac{1}{b}-L_E\right)-\alpha(\chi(L_B)-1)L_y^{-1}\left(1-\frac{1}{b}-\frac{L_E}{2}\right)L_E\right]\Bigg\} \ .$$

Rearranging terms yields:

$$\begin{aligned}\frac{\partial W}{\partial L_E} =& -(1-\alpha)L_y^{-\alpha}+(1-\alpha)L_y^{-\alpha}\chi(L_B)b(1-L_E) \\ & - t_P(1-\alpha)L_y^{-\alpha}\chi(L_B)b\left(1-\frac{1}{b}-L_E\right) \\ & -(1-t_P)\alpha(1-\alpha)L_y^{-1-\alpha}\chi(L_B)b(\chi(L_B)-1)\left(1-\frac{1}{b}-\frac{L_E}{2}\right)L_E \ . \end{aligned} \tag{27}$$

Equation (27) characterizes the trade-offs faced by the social planner when considering to marginally increase entrepreneurship in the economy. It reveals why tax neutrality, i.e. $t_L = t_P$, is not welfare maximizing in our economy in general.

The first summand represents the marginal product of labor used in final good production - which corresponds to the pre-tax wage in equilibrium, $(1-\alpha)L_Y^{-\alpha}$ - lost as the marginal entrepreneur is not available for the labor market anymore. $(1-\alpha)L_y^{-\alpha}\chi(L_B)b(1-L_E)$ is the pre-tax expected utility that this marginal entrepreneur can earn. Assume tax neutrality, then the first two summands exactly reflect the trade-off faced by the marginal entrepreneur and hence they cancel. To see this, note that under tax neutrality each potential entrepreneur k compares his pre-tax wage earned in the labor market, $(1-\alpha)L_Y^{-\alpha}$, with the pre-tax expected utility from being an entrepreneur, $(1-\alpha)L_y^{-\alpha}\chi(L_B)b(1-k)$. The result then follows from $k = L_E$ for the marginal entrepreneur.

By contrast, the remaining two summands in equation (27) are not 0 in general under tax neutrality. $-t_P(1-\alpha)L_y^{-\alpha}\chi(L_B)b\left(1-\frac{1}{b}-L_E\right)$ captures the immaterial utility of the marginal entrepreneur that is lost due to profit taxes. For the occupational choice of the marginal entrepreneur, only the relation of profit to labor income taxes matters, i.e. his choice would remain the same for any $t_L = t_P$. Furthermore, with regard to consumption, for a constant τ, t_L and t_P have purely distributional effects which do not matter for aggregate welfare in our economy. However, t_P does not only decrease expected after-tax profits of the marginal entrepreneur, but also his immaterial utility. This reduction in immaterial utility of the marginal entrepreneur is lost for aggregate welfare. It could be eliminated by having $t_L = t_P = 0$.

Still, the last summand would remain. This summand captures the effect of the marginal entrepreneur on equilibrium wages, which affects the immaterial utility earned by all other entrepreneurs. The sign of this effect depends on two different factors: First, on $1 - \frac{1}{b} - \frac{L_E}{2} \gtrless 0$ which determines whether this immaterial utility is positive or negative in aggregate. And second, on $\chi(L_B) - 1 \gtrless 0$ which determines whether the marginal entrepreneur has a positive or a negative effect on equilibrium wages. This term is not 0 in general for $t_L = t_P = 0$.

In summary, we have argued that any given level τ should be implemented either in a t_P-minimizing or in a t_P-maximizing way and that tax neutrality is not optimal in general. Taken together, these two observations give rise to *pecking orders of taxation* and hence reinforce our main insights from the analysis of aggregate consumption maximizing policies. Proposition 11 establishes the welfare maximizing pecking orders formally, where (t_L^*, t_P^*, L_B^*) denote again optimal policy choices and L_E^* denotes the resulting equilibrium level of entrepreneurship in the economy.

Proposition 11 (Welfare Optimal Pecking Order of Taxation)
The welfare optimal tax policy for economies in which entrepreneurs are active can be characterized as follows:

(i) if $L_E^* < \min\left\{1 - \frac{1-\underline{t}_L}{(1-\underline{t}_P)\chi(L_B^*)b}, 2\left(1 - \frac{1}{b}\right)\right\}$, *then* $t_P^* > \underline{t}_P$ *and* $t_L^* = \underline{t}_L$;

(ii) if $1 - \frac{1-\underline{t}_L}{(1-\underline{t}_P)\chi(L_B^*)b} < L_E^* < 2\left(1 - \frac{1}{b}\right)$, *then* $t_P^* = \underline{t}_P$ *and* $t_L^* > \underline{t}_L$;

(iii) if $2\left(1 - \frac{1}{b}\right) < L_E^* < 1 - \frac{1-\bar{t}_L}{(1-\bar{t}_P)\chi(L_B^*)b}$, *then* $t_P^* = \bar{t}_P$ *and* $t_L^* < \bar{t}_L$;

(iv) if $L_E^* > \max\left\{1 - \frac{1-\bar{t}_L}{(1-\bar{t}_P)\chi(L_B^*)b}, 2\left(1 - \frac{1}{b}\right)\right\}$, *then* $t_P^* < \bar{t}_P$ *and* $t_L^* = \bar{t}_L$.

A proof that includes all cases, including knife-edge cases, is given in appendix B.

Cases (i) and (iii) of Proposition 11 give rise to a pecking order with profit taxes first in a sense that either t_L is at its lower constitutional bound and t_P is not or t_P is at its upper constitutional bound and t_L is not. Conversely, cases (ii) and (iv) give rise to a pecking order with labor income tax first.

We note, however, that as opposed to the setting with no lump-sum taxes considered in section 5.3, the pecking order is not a result of the government seeking to raise additional funds in order to finance basic research once the preferred tax measure cannot be relied upon further. Optimal tax policies are rather driven by the endeavor to implement any preferred τ either in a t_P-maximizing or in a t_P-minimizing way, as discussed above. In cases (i) and (ii) of Proposition 11, for example, the aggregate extra (dis)-utility of entrepreneurs is positive ($L_E^* < 2\left(1 - \frac{1}{b}\right)$) and hence the government seeks to have a minimal t_P in order not to lose this extra utility and primarily uses t_L to induce the desired level of entrepreneurship. If entrepreneurship is desirable from a social welfare perspective, the government opts for $t_L^* > \underline{t}_L$ to incentivize entrepreneurship (case (ii)). If entrepreneurial activity becomes less attractive, the government first responds by decreasing t_L to discourage entrepreneurship and once t_L cannot be relied upon any further because it reached its lower constitutional bound, it increases t_P thereby trading-off the social welfare gain from further discouraging entrepreneurship against the cost of losing some of the extra utility earned by entrepreneurs (case (i)). As a side-effect, the pecking orders derived here are solely characterized by constitutional bounds of taxation and in particular peaks of the Laffer Curves which played a central role in the pecking orders derived in section 5.3 do not matter.

We further note that for the two cases that yield the same pecking order according to Proposition 11 the underlying motives are different. Consider for example case (iii) of Proposition 11 as opposed to case (i) which both motivate a pecking order with profit taxes first. Here, the aggregate extra (dis)-utility term of entrepreneurs is negative ($L_E^* > 2\left(1 - \frac{1}{b}\right)$) and hence the government chooses $t_P^* = \bar{t}_P$ in order to minimize these welfare losses for any given level L_E. In addition, it uses t_L to further discourage entrepreneurship and hence chooses $t_L < \bar{t}_L$.

Finally, it is important to note that albeit the just discussed differences between the pecking orders identified, they share the same fundamental motive: The pecking order with profit taxes first is preferable whenever the desired level of entrepreneurship is relatively low. By contrast, the pecking order with labor income tax first is preferable whenever the desired level of entrepreneurship is relatively high. In the setting considered here, a relatively high level of entrepreneurial activity refers to:

- a level larger than the one implied by $t_L = \underline{t}_L$ and $t_P = \underline{t}_P$ if aggregate immaterial utility from entrepreneurship is positive (case (ii));
- a level larger than the one implied by $t_L = \bar{t}_L$ and $t_P = \bar{t}_P$ if aggregate immaterial utility from entrepreneurship is negative (case (iv)).

We summarize these qualitative results in the following Corollary:

Corollary 3
Suppose the government maximizes aggregate welfare, equation (26), *using* (t_L, t_P, t_H, L_B)

as policy instruments. Then:

(i) If the welfare optimal level of entrepreneurial activity is relatively high, then the government opts for the pecking order with labor income tax first.

(ii) If the welfare optimal level of entrepreneurial activity is relatively low, then the government opts for the pecking order with profit tax first.

The welfare optimal level of entrepreneurial activity depends on a variety of different factors. In particular, it depends on the effectiveness of entrepreneurship in terms of labor saved in intermediate good production, $\chi(L_B^*)$, and on the immaterial benefits from entrepreneurship as determined by b.

Proposition 11 limits attention to economies in which entrepreneurs are active, i.e. $L_E > 0$. Economically, this is not very restrictive for the purpose of our analysis as in an economy where $L_E^* = 0$, trivially $L_B^* = 0$ combined with any tax policy ensuring that $L_E^* = 0$ would be welfare maximizing. Proposition 12 analyzes when $L_E^* > 0$ is welfare optimizing for L_B given. Whether or not $L_E^* > 0$ is only interesting for cases where $L_E = 0$ and $L_E > 0$ are both feasible and hence attention is limited to these cases.[41]

Proposition 12

Suppose that $L_B = L_B^*$ and let $L_E = 0$ and $L_E > 0$ both be feasible. Then $L_E^* > 0$, i.e. for L_B^* given the welfare maximizing tax policy is one that yields positive entrepreneurship, if

$$\chi(L_B^*) > \frac{1}{1 + (1 - \tilde{t}_P)(b - 1)} \,, \tag{28}$$

where

$$\tilde{t}_P = \begin{cases} \min\left(\bar{t}_P, 1 - \frac{1-\bar{t}_L}{\chi(L_B^*)b}\right) & \text{if } b \leq 1 \\ \max\left(\underline{t}_P, 1 - \frac{1-\underline{t}_L}{\chi(L_B^*)b}\right) & \text{if } b > 1 \end{cases} . \tag{29}$$

A proof of Proposition 12 is given in appendix A.10. Proposition 12 implies quite intuitively that $L_E^* > 0$ is welfare optimal whenever $\chi(L_B^*)$ is large, i.e. whenever the expected labor saved for final good production from increasing the number of entrepreneurs is large.

[41] We note that in our model feasibility of a given level L_E does not only require the existence of a combination of tax measures t_L and t_P that yield the desired level of entrepreneurial activity given L_B, but also that this results in non-negative profits of the final good producer.

8 Conclusions

We have outlined a rationale for a pecking order of taxation to finance basic research investments, thus presenting an important new perspective on the theory of optimal income taxation. Moreover, we have characterized the conditions under which the optimal taxation scheme is politically viable. In particular, our political economy analysis suggests that optimal policies might harm workers, if they are not engaged in the stock-market. We have shown that an entrepreneurship and innovation stimulating policy might therefore not be politically viable if these workers formed a majority of the population. Given its importance for future economic growth and prosperity, further analyses might take on different perspectives on the political economy of financing basic research. For example, it may be interesting to analyze optimal policies from the point of view of entrepreneurs or shareholders. Also, possible ways of compensating workers for resulting welfare losses deserve further scrutiny. On a similar note, our analysis of optimal financing of basic research investments might also be further linked to the theory on optimal taxation in the tradition of Mirrlees (1971). With concave utilities and traditional supply side effects of labor income taxation, optimal policies would account for losses in aggregate utility from income inequality and for potential adverse effects on labor supply. These additional equity- / efficiency trade-offs might push optimal tax policies towards a more egalitarian economy, thus stimulating political support for welfare optimal policies. In the presence of incomplete markets, concave utilities might also allow for additional beneficial effects of basic research on entrepreneurship and thus innovation in the economy: next to fostering expected profits from being entrepreneur, basic research affects associated idiosyncratic risks.

A Proofs

A.1 Proof of Lemma 1

We prove each part of Lemma 1 in turn.

(i) We consider innovative and non-innovative intermediate good producer separately.

Intermediate goods in non-innovative industries are produced using the freely available standard technology. Perfect competition implies that these intermediate goods are sold at cost in equilibrium, i.e. non-innovative intermediate good producer will offer their goods at price $p(i) = mw$.

The cost of production of innovative intermediate good producer are reduced to γmw. These firms are still confronted with competition from non-innovative intermediate good producers in their industry. Taken together, this implies that an innovative intermediate good producer will charge a price $p(i) = \delta_i mw$ with $\delta_i \in [\gamma, 1]$. We now show by contradiction that $\delta_i \in [\gamma, 1)$ cannot be optimal. We show that there do not exist symmetric equilibria such that all innovative intermediate good producer charge the common price $p(i) = \delta mw$, with $\delta \in [\gamma, 1)$ and leave it to the reader to verify that no non-symmetric equilibrium exists with $\delta_i < 1$ for some i.

Let us define $\tilde{X} := \int_{i|p(i)=\delta mw} x(i)^\alpha di$ and $\hat{X} := \int_{i|p(i)=mw} x(i)^\alpha di$. This allows us to write the maximization problem of the final good producer as

$$\begin{aligned}\max_{L_y, X} \; \pi_y &= L_y^{1-\alpha}(\tilde{X} + \hat{X}) - wL_y - \delta mw\tilde{X} - mw\hat{X} \\ &= \tilde{X}(L_y^{1-\alpha} - \delta mw) + \hat{X}(L_y^{1-\alpha} - mw) - wL_y. \end{aligned} \tag{30}$$

$\delta < 1$ implies that $L_y^{1-\alpha} - \delta mw > 0$ is a necessary condition for the final good producer to operate making non-negative profits. $L_y^{1-\alpha} - \delta mw$ is the net marginal benefit of the final good producer from using intermediate good $x(i)$ offered at price $p(i) = \delta mw$ in production. Hence, $L_y^{1-\alpha} - \delta mw > 0$ implies first, that if the final good producer is operating he always demands $x(i) = 1$ of every intermediate offered at price $p(i) = \delta mw$. And second, that the innovative intermediate good producer i would want to set a price $\tilde{p}(i) = \delta mw + \epsilon$, $\epsilon > 0$ but small, such that $L_y^{1-\alpha} - \tilde{p}(i) > 0$. Then the net marginal benefit of the final good producer from using intermediate good $x(i)$ in production remains positive. Furthermore, given that each intermediate good producer has measure 0, it would not affect the profitability of the representative final good firm.

Hence, the final good firm would still demand $x(i) = 1$, a contradiction to $p(i) = \delta mw$ being profit maximizing for intermediate good producer i.

The contradiction establishes the result.

(ii) Let us define $X := \int_0^1 x(i)^\alpha di$. X assumes the value 0 if $x(i) = 0\ \forall i$, 1 if $x(i) = 1\ \forall i$, and values between 0 and 1 only if a subset of the varieties is used. If $p_i = mw\ \forall i$, the maximization problem of the final good producer can be written as

$$\max_{L_y, X}\ \pi_y = L_y^{1-\alpha} X - wL_y - mwX = X(L_y^{1-\alpha} - mw) - wL_y. \tag{31}$$

Hence, the profit function is linear in X. A necessary condition for non-negative profits is $L_y^{1-\alpha} - mw > 0$. As a consequence, if it is optimal for the final good producer to operate, i.e. to demand $X > 0$ then it must hold that $L_y^{1-\alpha} - mw > 0$ and hence profits are maximized by setting $X = 1$.

A.2 Proof of Proposition 1

From Lemma 1 and the expositions in the main text, we know that if condition (PPC) is satisfied, the final good producer is operating and he uses all varieties in production. Conversely, if condition (PPC) is not satisfied, he is not operating and $L_E^e = L_x^e = L_y^e = 0$ and zero profits follow immediately. It remains to show that in case (i) the other variables take on the unique equilibrium values stated in the Proposition.

(i) Conditions (1), (2), and (4), and (7) have been derived in the main text. Condition (3) follows from using L_E^e and L_x^e in the labor market clearing condition. Combining w^e with the observation that $p(i) = mw\ \forall i$ yields condition (5). Condition (6) follows from $x(i) = 1\ \forall i$ and the production technology in the final good sector. Finally, condition (8) follows from using w^e in the expression for profits of a monopolistic intermediate good producer.

A.3 Proof of Corollary 1

By Proposition 3 there will be an entrepreneurial economy if and only if condition (PLS) is satisfied. Now, in response to a change in m, b, τ, or γ, the government could leave $\tilde{L}_B(\tau)$ unaffected. Hence, if it opts for a $\hat{\tilde{L}}_B(\tau) \neq \tilde{L}_B(\tau)$, then we must have

$c\left(\tau, \hat{\tilde{L}}_B(\tau)\right) \geq c\left(\tau, \tilde{L}_B(\tau)\right)$, which implies

$$
\begin{aligned}
&-\hat{\tilde{L}}_B(\tau) + \left[1 - \frac{1}{\tau\eta\left(\hat{\tilde{L}}_B(\tau)\right)m(1-\gamma)b}\right]\left[\eta\left(\hat{\tilde{L}}_B(\tau)\right)m(1-\gamma)-1\right] \geq \\
&-\tilde{L}_B(\tau) + \left[1 - \frac{1}{\tau\eta\left(\tilde{L}_B(\tau)\right)m(1-\gamma)b}\right]\left[\eta\left(\tilde{L}_B(\tau)\right)m(1-\gamma)-1\right] .
\end{aligned}
$$

A proof then follows from the fact that for a constant $\tilde{L}_B(\tau)$

$$
\left[1 - \frac{1}{\tau\eta\left(\tilde{L}_B(\tau)\right)m(1-\gamma)b}\right]\left[\eta\left(\tilde{L}_B(\tau)\right)m(1-\gamma)-1\right]
$$

is increasing in m, b, and τ and decreasing in γ.

A.4 Proof of Proposition 4

To prove Proposition 4, it remains to show that $\overline{\tau}_O$ and $\underline{\tau}_O$ correspond to the pecking orders of taxation as described in the main text. We proof Proposition 4 (i) by contradiction. Part (ii) can be shown using a similar argument.

(i) We first note that $L_B > L_{B,min}$ implies that if $\left(\hat{t}_L, \hat{t}_P, \hat{L}_B\right)$ satisfies condition (PPC), then so does any $\left(t'_L, t'_P, \hat{L}_B\right)$ satisfying $\frac{1-t'_P}{1-t'_L} \geq \frac{1-\hat{t}_P}{1-\hat{t}_L}$.

Let $TR(t_L, t_P, L_B)$ denote tax revenues in working hour equivalents given t_L, t_P, and L_B. Consider a policy choice $\left(\hat{t}_L, \hat{t}_P, \hat{L}_B\right)$, such that $\hat{t}_P > 0$, $\hat{L}_B > L_{B,min}$, and $\exists$ $\hat{\hat{\tau}}_L > \hat{t}_L$ such that $TR\left(\hat{\hat{\tau}}_L, \hat{t}_P, \hat{L}_B\right) > TR\left(\hat{t}_L, \hat{t}_P, \hat{L}_B\right)$. Furthermore, let $\left(\hat{t}_L, \hat{t}_P, \hat{L}_B\right)$ satisfy condition (PPC). Then, by continuity of TR in t_L and t_P, it is possible to finance $\hat{L}_B$ using some alternative financing scheme (t'_L, t'_P) satisfying:

$$
\begin{aligned}
t'_L &= \hat{t}_L + \Delta_1 , \quad \Delta_1 \geq 0\text{, but small such that } t'_L \leq \bar{t}_L \\
t'_P &= \hat{t}_P - \Delta_2 , \quad \Delta_2 \geq 0\text{, but small such that } t'_P \geq 0 \\
\frac{1-t'_P}{1-t'_L} &> \frac{1-\hat{t}_P}{1-\hat{t}_L} .
\end{aligned}
$$

In particular, depending on whether $\frac{\partial TR}{\partial t_L} \lesseqgtr 0$ and $\frac{\partial TR}{\partial t_P} \lesseqgtr 0$, the following alternative financing schemes satisfy the conditions above:

1. Suppose $\left.\frac{\partial TR}{\partial t_L}\right|_{\substack{t_L=\hat{t}_L\\ t_P=\hat{t}_P\\ L_B=\hat{L}_B}} < 0$ or $\left(\left.\frac{\partial TR}{\partial t_L}\right|_{\substack{t_L=\hat{t}_L\\ t_P=\hat{t}_P\\ L_B=\hat{L}_B}} = 0 \text{ and } \left.\frac{\partial^2 TR}{(\partial t_L)^2}\right|_{\substack{t_L=\hat{t}_L\\ t_P=\hat{t}_P\\ L_B=\hat{L}_B}} < 0\right)$. Then by the existence of $\hat{\hat{\tau}}_L > \hat{t}_L$ such that $TR\left(\hat{\hat{\tau}}_L, \hat{t}_P, \hat{L}_B\right) > TR\left(\hat{t}_L, \hat{t}_P, \hat{L}_B\right)$ and by continuity of TR in t_L $\exists$ a $t'_L > \hat{t}_L$ satisfying $TR\left(t'_L, \hat{t}_P, \hat{L}_B\right) = TR\left(\hat{t}_L, \hat{t}_P, \hat{L}_B\right)$. We conclude that $\exists\ \Delta_1 > 0$ and $\Delta_2 = 0$ satisfying the conditions stated above.

2. Suppose $\left.\frac{\partial TR}{\partial t_P}\right|_{\substack{t_L=\hat{t}_L\\ t_P=\hat{t}_P\\ L_B=\hat{L}_B}} < 0$ or $\left(\left.\frac{\partial TR}{\partial t_P}\right|_{\substack{t_L=\hat{t}_L\\ t_P=\hat{t}_P\\ L_B=\hat{L}_B}} = 0 \text{ and } \left.\frac{\partial^2 TR}{(\partial t_P)^2}\right|_{\substack{t_L=\hat{t}_L\\ t_P=\hat{t}_P\\ L_B=\hat{L}_B}} > 0\right)$. We show that given $\hat{t}_L$ and $\hat{L}_B$, TR is minimized at $t_P = 0$. Then it follows from continuity of TR in t_P that $\exists$ a $t'_P < \hat{t}_P$ satisfying $TR\left(\hat{t}_L, t'_P, \hat{L}_B\right) = TR\left(\hat{t}_L, \hat{t}_P, \hat{L}_B\right)$. Hence, $\exists\ \Delta_1 = 0$ and $\Delta_2 > 0$ satisfying the conditions stated above.
   To show that given $\hat{t}_L$ and $\hat{L}_B$, TR is minimized at $t_P = 0$, note first that L_E is non-increasing in t_P. Hence, the term $(\bar{L}-L_E)t_L$ is non-decreasing in t_P. Furthermore, all $t_P < \hat{t}_P$ satisfy condition (PPC) and hence we have $t_P\left[\frac{\alpha}{1-\alpha}L_y - m + L_E\chi(L_B)\right] \geq 0$. We conclude that TR is minimized at $t_P = 0$.

3. Finally, suppose $\left.\frac{\partial TR}{\partial t_L}\right|_{\substack{t_L=\hat{t}_L\\ t_P=\hat{t}_P\\ L_B=\hat{L}_B}} > 0$ or $\left(\left.\frac{\partial TR}{\partial t_L}\right|_{\substack{t_L=\hat{t}_L\\ t_P=\hat{t}_P\\ L_B=\hat{L}_B}} = 0 \text{ and } \left.\frac{\partial^2 TR}{(\partial t_L)^2}\right|_{\substack{t_L=\hat{t}_L\\ t_P=\hat{t}_P\\ L_B=\hat{L}_B}} > 0\right)$
   and
   $\left.\frac{\partial TR}{\partial t_P}\right|_{\substack{t_L=\hat{t}_L\\ t_P=\hat{t}_P\\ L_B=\hat{L}_B}} > 0$ or $\left(\left.\frac{\partial TR}{\partial t_P}\right|_{\substack{t_L=\hat{t}_L\\ t_P=\hat{t}_P\\ L_B=\hat{L}_B}} = 0 \text{ and } \left.\frac{\partial^2 TR}{(\partial t_P)^2}\right|_{\substack{t_L=\hat{t}_L\\ t_P=\hat{t}_P\\ L_B=\hat{L}_B}} < 0\right)$.
   Then by continuity of TR in t_L and t_P $\exists$ a $t'_L > \hat{t}_L$ and $t'_P < \hat{t}_P$ satisfying $TR\left(t'_L, t'_P, \hat{L}_B\right) = TR\left(\hat{t}_L, \hat{t}_P, \hat{L}_B\right)$. We conclude that $\exists\ \Delta_1 > 0$ and $\Delta_2 > 0$ satisfying the conditions stated above.

$\frac{1-t'_P}{1-t'_L} > \frac{1-\hat{t}_P}{1-\hat{t}_L}$ implies $L'_E > \hat{L}_E$.[42] Since $\hat{L}_B > L_{B,min}$ and hence $\eta(\hat{L}_B)m(1-\gamma) > 1$ it follows $L'_y > \hat{L}_y$, a contradiction to $\left(\hat{t}_L, \hat{t}_P, \hat{L}_B\right)$ being optimal.

The contradiction establishes the result.

[42] We note that it can never be optimal to finance $L_B > 0$ in a way yielding $L_E = 0$.

A.5 Proof of Lemma 2

We first show the continuity of I in τ for given L_B and then the continuity of I in L_B for any given τ.

(1) Since the median voter's gross income is a continuous function of τ and L_B, it is sufficient to focus on net transfers $NT(\tau, L_B)$.

(2) Regarding the different cases of optimal profit and labor taxes for given (τ, L_B) as shown in Table 6.1, the net transfers are continuous within each of the different subsets of (τ, L_B) defined by the four different cases. Potential discontinuities may exist at the transition from one case to another. In this respect, we define the critical values $\tau^c(L_B)$ and $L_B^c(\tau)$ by $DNT(\tau^c, L_B) = 0$ for any given L_B in the feasible set and $DNT(\tau, L_B^c) = 0$.

(3) As can be observed in Table 6.1, there are two critical values of τ for a given L_B: $\tau^c(L_B)$ and $\tau = 1$. The former is only interesting if $\tau^c(L_B) \in [\underline{\tau}, \bar{\tau}]$, while the latter will always be in the feasible set by our assumptions in Section 3. Now consider any two sequences $\{\tau_n\}$ and $\{\tau_k\}$ with $\lim_n \tau_n = \tau_c$, $\tau_n \leq \tau_c$ and $\lim_k \tau_k = \tau_c$, $\tau_k \geq \tau_c$. As $DNT(\tau^c, L_B) = 0$ means that a change in tax rates t_P, t_L does not affect net transfers $[NT(\tau^c, L_B)]$ as long as τ^c remains unchanged, we must obtain $\lim_n NT(\tau_n, L_B) = \lim_k NT(\tau_k, L_B)$. Hence, $NT(\tau, L_B)$ is continuous at τ^c for a given L_B.

(4) At the critical value $\tau = 1$, both tax rates t_P and t_L are identical. Consequently, for two sequences with $\lim_n \tau_n = 1$, $\tau_n \leq 1$ and $\lim_k \tau_k = 1$, $\tau_k \geq 1$, we also obtain $\lim_n NT(\tau_n, L_B) = \lim_k NT(\tau_k, L_B) = NT(1, L_B)$. Thus, net transfers are continuous in τ at $\tau = 1$.

(5) We can use the same argument as in (3) with respect to sequences $\{L_{B,n}\}$ and $\{L_{B,k}\}$ with limit L_B^c for given τ.

A.6 Proof of Proposition 6

Consider some entrepreneurial economy which is aggregate consumption improving over the stagnant economy. We show that $I^E - I^S$ is strictly increasing in s. Then, if a worker with shares $s^\star$ prefers the entrepreneurial economy over the stagnant economy, so do all workers with shares $s \geq s^\star$. The result for a potential entrepreneur with shareholdings $s \geq s^\star$ then follows from the fact that he is a worker in the stagnant economy and that he is free to stay worker in the entrepreneurial economy.

Subtracting equation (24) from equation (23) and differentiating with respect to s we get:

$$\frac{d}{ds}\left[I^E - I^S\right] = w^E(1-t_P^E)\left(\frac{\alpha}{1-\alpha}l_y^E - l_m\right) - w^S(1-t_P^S)\left(\frac{\alpha}{1-\alpha}l_y^S - l_m\right) .$$

We have $1 > t_P^S = \bar{t}_P \geq t_P^E$. Furthermore, as the entrepreneurial economy is aggregate consumption improving over the stagnant economy and final good producer's profits are increasing in L_y it holds that

$$w^E\left(\frac{\alpha}{1-\alpha}l_y^E - l_m\right) > w^S\left(\frac{\alpha}{1-\alpha}l_y^S - l_m\right)$$

and hence

$$\frac{d}{ds}\left[I^E - I^S\right] > 0 .$$

This completes the proof.

A.7 Proof of Proposition 7

Note that income per capita can be written as

$$\bar{y} = \frac{y}{\bar{L}} = w\left[1 + \frac{\alpha}{1-\alpha}l_Y - l_m + l_E(\chi(L_B) - 1) - l_B\right], \tag{32}$$

which reduces to

$$\bar{y}^S = \frac{y^S}{\bar{L}} = w^S\left[1 + \frac{\alpha}{1-\alpha}l_Y^S - l_m\right], \tag{33}$$

in the stagnant economy. The median voter's income in the entrepreneurial economy and the stagnant economy are given in equations (23) and (24), respectively. Due to the assumption $s < 1$, the median voter maximally redistributes profits $t_P = \bar{t}$ in the stagnant economy.[43]

Consider any policy $(\hat{\tau}, \hat{L}_B)$ for which $\bar{y} > \bar{y}^S$ (such a policy necessarily implies $L_B > 0$ and $L_E > 0$). With $s < 1$, we have that $I^S \leq \bar{y}^S$. Hence it suffices to show that for $(\hat{\tau}, \hat{L}_B)$, we can find a $\bar{t}$ such that $I^E(\hat{\tau}, \hat{L}_B) > \bar{y}^S$. Note that $\lim_{t_P \to 1, t_l \to 1} I^E = \bar{y}$. Since $\bar{y}(\hat{\tau}, \hat{L}_B) > \bar{y}^S$, the desideratum follows from the fact that for any $\delta > 0$, we can find a pair $(t_P, t_L) < (1,1)$ yielding $\hat{\tau}$ and

$$\bar{y}(\hat{\tau}, \hat{L}_B) - I^E(\hat{\tau}, \hat{L}_B) \leq \delta.$$

This completes the proof.

[43]Note that the labor tax does not affect the median voter's income in the stagnant economy as all individuals are workers. The population only differs with respect to stocks of the final good firm.

A.8 Proof of Proposition 8

To show the result, note first that the restriction to $s \leq \frac{\bar{L}}{\bar{L}+(1-\gamma)m}$ is a sufficient condition for a negative derivative of the median voter's gross income with respect to L_y. The value of $s \leq \frac{\bar{L}}{\bar{L}+(1-\gamma)m}$ follows from the fact that $L_y < \bar{L} - \gamma m$.

Now, suppose that $\bar{t} = 0$. Then, the median voter's income corresponds to his gross income minus his share in the cost for providing basic research and he strictly prefers the stagnant economy over the entrepreneurial economy.[44] The result then follows from the continuity of the median voter's income, implying that he will also prefer the stagnant economy for sufficiently small $\bar{t} > 0$.

A.9 Proof of Proposition 9

Fix any entrepreneurial policy $(\hat{\tau}, \hat{L}_B)$ with $\hat{L}_y \geq L_y^S$. From Proposition 7 we know that for $\bar{t} \to 1$ the following two conditions are satisfied:

1. $\hat{\tau} \in [\underline{\tau}, \overline{\tau}]$,
2. the median voter with $s \leq \frac{\bar{L}}{\bar{L}+(1-\gamma)m}$ will prefer the entrepreneurial policy $(\hat{\tau}, \hat{L}_B)$ over the stagnant economy.

From Proposition 8 we know that for $\bar{t}$ small the median voter supports the stagnant economy, implying that at least one of the two conditions above is no longer satisfied. Hence, it remains to show that for every entrepreneurial policy $(\hat{\tau}, \hat{L}_B)$ $\exists$ a unique threshold level $\bar{t}_c$ such that both conditions above are satisfied if and only if $\bar{t} \geq \bar{t}_c$.

For every $\hat{\tau} \in (0, 1)$ $\exists$ a unique $\bar{t}_c^1$ such that $\hat{\tau} \in [\underline{\tau}, \overline{\tau}]$ if and only if $\bar{t} \geq \bar{t}_c^1$. Hence, we can limit attention to $\bar{t} \geq \bar{t}_c^1$ and the result follows from showing that $\hat{I}^E - I^S$ is monotonous in $\bar{t}$. Note that a decrease in $\bar{t}$ such that $\bar{t} \geq \bar{t}_c^1$ will only change net transfers but not the median voter's gross income. Hence, we can limit attention to the derivative of NT with respect to $\bar{t}$ for $\hat{\tau}$ and $\hat{L}_B$ given. In the stagnant economy we have:

$$\left.\frac{\partial NT^S}{\partial \bar{t}}\right|_{\substack{\hat{\tau} \\ \hat{L}_B \\ \bar{t} \geq \bar{t}_c^1}} = w^S \left[\left(\frac{\alpha}{1-\alpha} l_Y^S - l_m \right) (1-s) \right] \geq 0 \; .$$

[44] Note that $L_y \geq L_y^S$ and $L_B > 0$ in the entrepreneurial economy.

Note that $\frac{\partial NT^S}{\partial \bar{t}}$ is constant. The monotonicity of $\hat{I}^E - I^S$ then follows from $\frac{\partial NT^E}{\partial \bar{t}}$ being constant as well which we show to hold for each of the four cases outlined in table 6.1 separately.

$\boldsymbol{DNT < 0, \hat{\tau} \geq 1}$ Not possible as $\hat{L}_y \geq L_y^S$ implies $\chi(\hat{L}_B) > 1$ and $s \leq \frac{\bar{L}}{\bar{L}+(1-\gamma)m} < 1$.

$\boldsymbol{DNT < 0, \hat{\tau} < 1}$ The median voter optimally chooses $\hat{t}_L = \underline{t} = 0$ and $\hat{t}_P = 1 - \hat{\tau}$ implying that

$$\left.\frac{\partial NT^E}{\partial \bar{t}}\right|_{\substack{\hat{\tau} \\ \hat{L}_B \\ \bar{t} \geq \bar{t}_c^1}} = 0$$

and hence $\hat{I}^E - I^S$ is monotonous in $\bar{t}$.[45]

$\boldsymbol{DNT \geq 0, \hat{\tau} \geq 1}$ The median voter optimally chooses $\hat{t}_L = \bar{t}$ and $\hat{t}_P = 1 - \hat{\tau}(1 - \bar{t})$. Hence, the derivative of net transfers in the entrepreneurial economy with respect to $\bar{t}$ writes

$$\left.\frac{\partial NT^E}{\partial \bar{t}}\right|_{\substack{\hat{\tau} \\ \hat{L}_B \\ \bar{t} \geq \bar{t}_c^1}} = w^E \left[\hat{\tau}\left((\frac{\alpha}{1-\alpha} l_Y^E - l_m)(1-s) + \chi(\hat{L}_B) l_E\right) - l_E\right],$$

which is constant implying that $\hat{I}^E - I^S$ is monotonous in $\bar{t}$.[46]

$\boldsymbol{DNT \geq 0, \hat{\tau} < 1}$ The median voter optimally chooses $\hat{t}_L = 1 - (1 - \bar{t})/\hat{\tau}$ and $\hat{t}_P = \bar{t}$, yielding the following derivative of net transfers in the entrepreneurial economy

$$\left.\frac{\partial NT^E}{\partial \bar{t}}\right|_{\substack{\hat{\tau} \\ \hat{L}_B \\ \bar{t} \geq \bar{t}_c^1}} = w^E \left[\left((\frac{\alpha}{1-\alpha} l_Y^E - l_m)(1-s) + \chi(\hat{L}_B) l_E\right) - \frac{l_E}{\hat{\tau}}\right].$$

Again, $\frac{\partial NT^E}{\partial \bar{t}}$ is constant, implying that $\hat{I}^E - I^S$ is monotonous in $\bar{t}$.

A.10 Proof of Proposition 12

For $L_E = 0$, W does not depend on the choice of t_L and t_P. Hence, $L_E > 0$ is optimal if $\exists$ a tax policy, $\hat{t}_L$ and $\hat{t}_P$ such that L_E is just equal to 0, i.e. $1 - \frac{1-\hat{t}_L}{(1-\hat{t}_P)\chi(L_B^*)b} = 0$, and $\left.\frac{\partial W}{\partial L_E}\right|_{\substack{t_L = \hat{t}_L, \\ t_P = \hat{t}_P}} > 0$. In what follows, we show that this is the case if and only if the condition stated in Proposition 12 is satisfied.

[45]Note that $\frac{\partial NT^E}{\partial \bar{t}} = 0$, $\frac{\partial NT^S}{\partial \bar{t}} \geq 0$ and Proposition 7 imply that in the case considered here the median voter will prefer the entrepreneurial economy over the stagnant economy whenever feasible, i.e. we have $\bar{t}_c = \bar{t}_c^1 = 1 - \hat{\tau}$.

[46]In fact, we have $\bar{t}_c > \bar{t}_c^1$. This follows from $t_P = 0$ and hence $NT < 0$ for $\hat{\tau} = \overline{\tau}$.

Differentiating W with respect to L_E yields:

$$\begin{aligned}\frac{\partial W}{\partial L_E} =&(1-\alpha)L_y^{-\alpha}\Big\{(\chi(L_B^*)-1)+(1-t_P)\chi(L_B^*)b\\ &\left[\left(1-\frac{1}{b}-L_E\right)-\alpha(\chi(L_B^*)-1)L_y^{-1}\left(1-\frac{1}{b}-\frac{L_E}{2}\right)L_E\right]\Big\}\ .\end{aligned}$$

Evaluated at $L_E=0$, this reduces to:

$$\left.\frac{\partial W}{\partial L_E}\right|_{L_E=0} = (1-\alpha)(\bar{L}-L_B^*-m)\left[\chi(L_B^*)-1+(1-t_P)\chi(L_B^*)(b-1)\right]\ .$$

The non-negativity condition for profits of the final good producer combined with the feasibility of $L_E=0$ imply that $\bar{L}-L_B^*\geq\frac{m}{\alpha}$ and hence $(\bar{L}-L_B^*-m)>0$. We conclude:

$$\left.\frac{\partial W}{\partial L_E}\right|_{L_E=0} >0 \quad \text{if and only if} \quad \chi(L_B^*)>\frac{1}{1+(1-t_P)(b-1)}\ .$$

We notice that whether or not $\left.\frac{\partial W}{\partial L_E}\right|_{L_E=0}>0$ depends on the choice of t_P. In particular, for $(\bar{L}-L_B^*-m)>0$

$$\left.\frac{\partial W}{\partial L_E}\right|_{L_E=0} \text{ is } \begin{cases}\text{increasing in } t_P & \text{if } b<1\\ \text{independent of } t_P & \text{if } b=1\\ \text{decreasing in } t_P & \text{if } b>1\end{cases}\ .$$

We conclude that for $b\leq 1$, $\frac{\partial W}{\partial L_E}>0$ for some choice of t_L and t_P satisfying $1-\frac{1-t_L}{(1-t_P)\chi(L_B^*)b}=0$ if and only if $\chi(L_B^*)>\frac{1}{1+(1-t_P)(b-1)}$ for the biggest possible t_P satisfying $1-\frac{1-t_L}{(1-t_P)\chi(L_B^*)b}=0$. Conversely, if $b>1$, $\frac{\partial W}{\partial L_E}>0$ for some choice of t_L and t_P satisfying $1-\frac{1-t_L}{(1-t_P)\chi(L_B^*)b}=0$ if and only if $\chi(L_B^*)>\frac{1}{1+(1-t_P)(b-1)}$ for the smallest possible t_P satisfying $1-\frac{1-t_L}{(1-t_P)\chi(L_B^*)b}=0$. $\tilde{t}_P$ in condition (28) has been chosen accordingly.

B Welfare Maximizing Tax Policy

Proposition 11 in the main text characterized the welfare optimal tax policies for L_B given, where optimal tax policies were dependent on the level of entrepreneurial activity. Proposition 13 presents an extended version of this characterization, including also relevant knife-edge cases.

Proposition 13

The welfare optimal tax policy can be characterized as follows:

	Case			Tax Policy
1	$L_E^* > 0$	-	-	$t_P^* = \underline{t}_P$, $t_P^* = \bar{t}_P$, $t_L^* = \underline{t}_L$ or $t_L^* = \bar{t}_L$
1.1		$1 - \frac{1}{b} - \frac{L_E^*}{2} > 0$	-	$t_P^* = \underline{t}_P$ and / or $t_L^* = \underline{t}_L$
1.1.1			$L_E^* > 1 - \frac{1-\underline{t}_L}{(1-\underline{t}_P)\chi(L_B^*)b}$	$t_P^* = \underline{t}_P$ and $t_L^* > \underline{t}_L$
1.1.2			$L_E^* = 1 - \frac{1-\underline{t}_L}{(1-\underline{t}_P)\chi(L_B^*)b}$	$t_P^* = \underline{t}_P$ and $t_L^* = \underline{t}_L$
1.1.3			$L_E^* < 1 - \frac{1-\underline{t}_L}{(1-\underline{t}_P)\chi(L_B^*)b}$	$t_P^* > \underline{t}_P$ and $t_L^* = \underline{t}_L$
1.2		$1 - \frac{1}{b} - \frac{L_E^*}{2} = 0$	-	$t_P^* = \underline{t}_P$ and $t_L^* = \bar{t}_L$ or $t_P^* = \bar{t}_P$ and $t_L^* = \underline{t}_L$
1.3		$1 - \frac{1}{b} - \frac{L_E^*}{2} < 0$	-	$t_P^* = \bar{t}_P$ and / or $t_L^* = \bar{t}_L$
1.3.1			$L_E^* > 1 - \frac{1-\bar{t}_L}{(1-\bar{t}_P)\chi(L_B^*)b}$	$t_P^* < \bar{t}_P$ and $t_L^* = \bar{t}_L$
1.3.2			$L_E^* = 1 - \frac{1-\bar{t}_L}{(1-\bar{t}_P)\chi(L_B^*)b}$	$t_P^* = \bar{t}_P$ and $t_L^* = \bar{t}_L$
1.3.3			$L_E^* < 1 - \frac{1-\bar{t}_L}{(1-\bar{t}_P)\chi(L_B^*)b}$	$t_P^* = \bar{t}_P$ and $t_L^* < \bar{t}_L$
2	$L_E^* = 0$	-	-	any feasible t_L^*, t_P^* s.t. $\frac{1-t_L^*}{(1-t_P^*)\chi(L_B^*)b} \geq 1$

B.1 Proof of Proposition 13.1

Implied by Proposition 13.1.1-3.

B.2 Proof of Proposition 13.1.1

We prove the result by contradiction.

$0 < L_E < 2(1 - \frac{1}{b})$ implies that the immaterial utility of entrepreneurs in the aggregate welfare, $(1 - t_P)\chi(L_B)(1-\alpha)bL_y^{-\alpha}L_E\left[1 - \frac{1}{b} - \frac{L_E}{2}\right]$, is positive. Now, consider a policy choice $\left(\hat{t}_L, \hat{t}_P, \hat{L}_B\right)$ such that $\hat{t}_L > \underline{t}_L$, $\hat{t}_P > \underline{t}_P$ and $\chi(\hat{L}_B)(2-b) < \frac{1-\hat{t}_L}{1-\hat{t}_P} < \chi(\hat{L}_B)b$ which is equivalent to $0 < L_E < 2(1 - \frac{1}{b})$. Then the following deviation is feasible

$$
\begin{aligned}
t'_P &= \hat{t}_P - \Delta_1\,, \quad \Delta_1 > 0, \text{ but small such that } t'_P \geq \underline{t}_P \\
t'_L &= \hat{t}_L - \Delta_2\,, \quad \Delta_2 > 0, \text{ but small such that } t'_L \geq \underline{t}_L \\
L'_B &= \hat{L}_B\,,
\end{aligned}
$$

and where Δ_1 and Δ_2 are chosen to satisfy

$$\frac{1-\hat{t}_P}{1-\hat{t}_L} = \frac{1-t'_P}{1-t'_L} \; .$$

Then $L'_E = \hat{L}_E$, $L'_y = \hat{L}_y$, and hence $W(t'_L, t'_P, L'_B) > W(\hat{t}_L, \hat{t}_P, \hat{L}_B)$, a contradiction to $\left(\hat{t}_L, \hat{t}_P, \hat{L}_B\right)$ being a welfare optimum.

The contradiction establishes the result.

B.3 Proof of Proposition 13.1.1.1-3

Immediately follows from Proposition 13.1.1.

B.4 Proof of Proposition 13.1.2

We prove the result by contradiction.

Consider a policy choice $\left(\hat{t}_L, \hat{t}_P, \hat{L}_B\right)$, such that $0 < L_E = 2(1-\frac{1}{b})$ and where $\hat{t}_L$ and $\hat{t}_P$ are not located at opposing boundaries of their respective feasible sets. Then it must be possible to either increase or decrease both tax measures, t_L and t_P. Furthermore, for $L_E = 2(1-\frac{1}{b})$, the following relationship between the partial derivatives of W with respect to t_L, t_P, and L_E holds:

$$\frac{\partial W}{\partial t_P} = -\frac{\partial W}{\partial t_L}\frac{1}{\tau} = -\frac{\partial W}{\partial L_E}\frac{1-t_L}{(1-t_P)^2\chi(L_B)b} \; .$$

As a consequence, $\left.\frac{\partial W}{\partial L_E}\right|_{\substack{t_L=\hat{t}_L,\\ t_P=\hat{t}_P,\\ L_B=\hat{L}_B}} = 0$ is a necessary condition for $\left(\hat{t}_L, \hat{t}_P, \hat{L}_B\right)$ to be a welfare optimum. Using $\hat{L}_E = 2(1-\frac{1}{b})$, $\frac{\partial W}{\partial L_E}$ reduces to:

$$\left.\frac{\partial W}{\partial L_E}\right|_{\substack{t_L=\hat{t}_L,\\ t_P=\hat{t}_P,\\ L_B=\hat{L}_B}} = (1-\alpha)L_y^{-\alpha}\left[(\chi(\hat{L}_B)-1)-(1-\hat{t}_P)\chi(\hat{L}_B)(b-1)\right] \; . \tag{34}$$

Next, consider the following deviation:

$$\begin{aligned} t'_P &= \hat{t}_P + \Delta_1 \,, \quad \Delta_1 \neq 0, \text{ but small such that } \underline{t}_P \leq t'_P \leq \bar{t}_P \\ t'_L &= \hat{t}_L + \Delta_2 \,, \quad \Delta_2 \neq 0, \text{ but small such that } \underline{t}_L < t'_L < \bar{t}_L \\ L'_B &= \hat{L}_B \,, \end{aligned}$$

i.e. t'_L and t'_P are not located at opposing boundaries of there feasible sets, and where Δ_1 and Δ_2 are chosen to satisfy[47]

$$\frac{1-\hat{t}_P}{1-\hat{t}_L} = \frac{1-t'_P}{1-t'_L} \ .$$

Then $L'_E = \hat{L}_E$, $L'_y = \hat{L}_y$, and hence $W(t'_L, t'_P, L'_B) = W(\hat{t}_L, \hat{t}_P, \hat{L}_B)$, i.e. if $\left(\hat{t}_L, \hat{t}_P, \hat{L}_B\right)$ is a welfare optimum, so is (t'_L, t'_P, L'_B). Now, $\hat{L}_E = 2(1-\frac{1}{b}) > 0$ implies that $b > 1$. Hence, we know from equation (34) that if $\frac{\partial W}{\partial L_E}\Big|_{\substack{t_L=\hat{t}_L,\\ t_P=\hat{t}_P,\\ L_B=\hat{L}_B}} = 0$, then it must be $\frac{\partial W}{\partial L_E}\Big|_{\substack{t_L=t'_L,\\ t_P=t'_P,\\ L_B=L'_B}} \neq 0$, a contradiction to $\left(\hat{t}_L, \hat{t}_P, \hat{L}_B\right)$ being a welfare optimum.

The contradiction establishes the result.

B.5 Proof of Proposition 13.1.3

We prove the result by contradiction.

With $L_E > \max(0, 2(1-\frac{1}{b}))$ the immaterial utility of entrepreneurs in the aggregate welfare, $(1-t_P)\chi(L_B)(1-\alpha)bL_y^{-\alpha}L_E\left[1-\frac{1}{b}-\frac{L_E}{2}\right]$, is negative. Now, consider a policy choice $\left(\hat{t}_L, \hat{t}_P, \hat{L}_B\right)$ such that $\hat{t}_L < \bar{t}_L$ and $\hat{t}_P < \bar{t}_P$ and where $\frac{1-\hat{t}_L}{1-\hat{t}_P} < \min(\chi(\hat{L}_B)b, \chi(\hat{L}_B)(2-b))$ which is equivalent to $L_E > \max(0, 2(1-\frac{1}{b}))$. Then the following policy choice is feasible

$$\begin{aligned} t'_P &= \hat{t}_P + \Delta_1\,, \quad \Delta_1 > 0\text{, but small such that } t'_P \leq \bar{t}_P \\ t'_L &= \hat{t}_L + \Delta_2\,, \quad \Delta_2 > 0\text{, but small such that } t'_L \leq \bar{t}_L \\ L'_B &= \hat{L}_B \ , \end{aligned}$$

where Δ_1 and Δ_2 are chosen to satisfy

$$\frac{1-\hat{t}_P}{1-\hat{t}_L} = \frac{1-t'_P}{1-t'_L} \ .$$

[47]Formally, we need to rule out the special cases where $\frac{1-\hat{t}_P}{1-\hat{t}_L} = 0$ and where $\frac{1-\hat{t}_P}{1-\hat{t}_L}$ is not well defined, i.e. the cases $\hat{t}_P = \bar{t}_P = 1$ and $\hat{t}_L = \bar{t}_L = 1$. $\hat{t}_P = 1$ is not possible as it cannot satisfy $0 < L_E = 2(1-\frac{1}{b})$ which is equivalent to $\chi(\hat{L}_B)(2-b) = \frac{1-\hat{t}_L}{1-\hat{t}_P} < \chi(\hat{L}_B)b$. For $\hat{t}_L = 1$ to satisfy $\chi(\hat{L}_B)(2-b) = \frac{1-\hat{t}_L}{1-\hat{t}_P} < \chi(\hat{L}_B)b$ it must hold that $b = 2$. Then any $\hat{t}_P < 1$ would imply that $L_E = 1$ and the aggregate immaterial utility of entrepreneurs is 0. Hence, any $\hat{t}_P < 1$ would yield the same welfare level. This is a special case which is not very interesting economically. We assume that $\hat{t}_P = \underline{t}_P$ in such case.

Then $L'_E = \hat{L}_E$, $L'_y = \hat{L}_y$, and hence $W(t'_L, t'_P, L'_B) > W(\hat{t}_L, \hat{t}_P, \hat{L}_B)$, a contradiction to $\left(\hat{t}_L, \hat{t}_P, \hat{L}_B\right)$ being a welfare optimum.

The contradiction establishes the result.

B.6 Proof of Proposition 13.1.3.1-3

Immediately follows from Proposition 13.1.3.

B.7 Proof of Proposition 13.2

Immediately follows from observing that for $L^*_E = 0$ all tax policies satisfying $\frac{1-t_L}{(1-t_P)\chi(L^*_B)b} \geq 1$, i.e. yielding $L_E = 0$, result in the same aggregate welfare.

C List of Notations

Symbol	Meaning	Range
$\bar{L}$	measure of the population	$\bar{L} > 1$
k	index of agents	$k \in [0, \bar{L}]$
y	output of the final good	$y \geq 0$
c	consumption of the final good	
$u(c)$	utility from final consumption good	$u(c) = c$
C	Aggregate consumption of the final good	$C = y$
π_y	profit of representative final good producer	$\pi_y \geq 0$
L_y	labor used in final good production	$L_y \geq 0$
i	index of intermediate goods	$i \in [0, 1]$
$x(i)$	amount of intermediate good i	$x(i) \in \{0, 1\}$
α	productivity parameter for intermediates used in final good production	$\alpha \in (0, 1)$
m	labor needed for producing intermediate i using standard technology	$m > 0$
γ	efficiency gain from innovation	$0 < \gamma < 1$
$L_x(i)$	labor needed to produce $x(i)$	$L_x(i) \in \{0, m, m\gamma\}$
L_x	labor demand from all intermediate sectors	$0 \leq L_x \leq 1$
$p(i)$	price for intermediate good i	$p(i) \geq 0$
$\pi_{xm}(i)$	profit of intermediate monopolist firm producing i	$\pi_{xm}(i) \geq 0$
$\lambda(k)$	(dis-)utility factor of individual k from being an entrepreneur	$\lambda_k \geq 0$
b	parameter in λ_k	$b > 0$
L_B	measure of population employed in basic research	$0 \leq L_B \leq \bar{L}$
$\eta(L_B)$	innovation probability as a function of L_B	$\eta(0) \geq 0$, $\eta'(\cdot) > 0$, $\eta''(\cdot) < 0$ and $\eta(\bar{L}) \leq 1$

Symbol	Meaning	Range
L_E	measure of the population who decide to become entrepreneurs	$0 \leq L_E \leq 1$
$\chi(L_B)$	expected marginal reduction of labor used in intermediate goods production from marginally increasing L_E	$\chi(L_B) > 0$
t_L	tax rate on labor income	$0 \leq t_L \leq \bar{t}_L$
t_P	tax rate on profits	$0 \leq t_P \leq \bar{t}_P$
t_H	lump-sum tax	
π^E	expected net profit for an entrepreneur	$\pi^E = (1 - t_P)\eta(L_B)\pi_{xm}$
$EU^E(k)$	expected utility of entrepreneur k	
w	wage rate	$w \geq 0$
W	aggregate welfare	
TR	tax revenue in working hour equivalents	
NT	net transfers to the median voter	
I^E	median voter's income in the entrepreneurial economy	
I^S	median voter's income in the stagnant economy	

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