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Endogenous Growth Theory

Lecture Notes for the winter term 2010/2011

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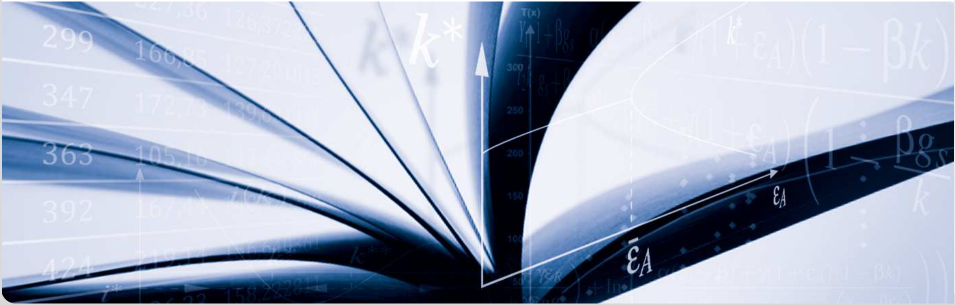

Introduction

- So far technology has not been endogenized, and growth has been either
  - exogenous as in the Solow model,
  - due to (linear) capital accumulation as in the AK Model or
  - occurred as a by-product of knowledge spillovers like in the Romer Model.
- This lecture provides some foundations for the models of endogenous technological change that are developed in the following lectures.

Types of Technological Change I

- One often distinguishes between
  - product innovations like the introduction of the first DVD player and
  - process innovations that reduce the costs of producing already existing goods.
- Concerning process innovations one might further differentiate between those that
  - improve the quality of existing products and
  - those that lead to cost reductions.
- However, in typical growth models these two innovations are often equivalent, since it is assumed that quantity and quality are perfect substitutes.

Types of Technological Change II

- Innovations are also often distinguished along the lines of macro and micro innovations.
  - Macro innovations are radical innovations and include general-purpose technologies like electricity, ICT, and nanotechnology.
  - Micro innovations on the other hand are more common, have a smaller impact on the economy, and include, for instance, innovations that lead to cost reductions.
- Empirical evidence indicates that micro innovations are responsible for most productivity growth.

A Production Function for Technology?

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- The objective is to model endogenous technological change.
- This implies that firm or individuals must be able to choose between different types of technologies.
- Through their choice of
  - research spending
  - effort,
  - and investment

  they should be able to invent better technologies.
- An implication of this argument is that there should exist a production function over production functions which relates these inputs to (possibly stochastic) output in the form of new technologies.
- Other names for this function are “meta production function”, “R&D production function” or more commonly “innovation possibilities frontier”.

Nonrivalry of Ideas I

- The concept of nonrivalry of idea features particularly strong in the work of Paul Romer.
- What does this concept entail?
- Ideas are fundamentally different from other inputs like capital and labor.
- A worker or machine that is put to work in one specific plant, cannot be employed at a different factory at the same time.
- Labor and capital are rival production factors.
- One the other hand, an efficiency-increasing idea can be used by multiple producers at the same time, i.e. ideas are nonrival.

Nonrivalry of Ideas II

- Let us look at the production function to get a better understanding of the concept of nonrivalry of ideas.
- In the preceding models we have adopted a production function of the form $F(K, L, A)$ which exhibited constant returns to scale in capital and labor.
- How was this justified?
- $\Rightarrow$ Replication argument.
- If one has double the amount of capital and labor at hand, one should be able to just set up an identical factory at a different location.
- In the absence of externalities and given that land is not a restricting factor, the new factory should be able to produce just as much output as the old one.

Nonrivalry of Ideas III

- Romer's idea now is that if one endogenizes A and combines this with the concept of nonrivalry of ideas, this leads to *increasing returns to scale* in K, L, and A.
- What is the intuition behind this argument?
- If A were like the other two input factors, then for the replication argument from above to hold, in addition to K and L one would have to replicate A as well.
- $\Rightarrow$ Constant returns to scale in the three input factors if this were the case.
- However, if A already exists and is available for all producers, then there is no need to replicate it.
- The nonrivalry of ideas thus leads to *increasing returns to scale*.

Nonrivalry of Ideas IV

- The *market size effect* is also implied by the nonrivalry of ideas.
- The size of the potential market for an idea is important in determining whether to implement the idea (is it profitable to do so?) and whether to research it in the first place.
- Watt's business partner is quoted in Scherer (1984, p. 13): "It is not worth my while to manufacture your engine for three countries only, but I find it very worth my while to make it for all the world."
- The fact that an idea, once developed, can be used as often as one wishes at no additional cost, is the key characteristic that makes the market size effect so important.
- It is however crucial to keep innovations apart from *pure public goods*.
- The latter are not only nonrival, but also nonexcludable.
- Innovations may be excludable via e.g. patenting.

The Role of Profits in Technological Change

- What drives technological change?
- Scientific constraints that lead to breakthroughs or the profit motive?
- Historians place great emphasis on the former view, whereas most economists favor the profit motive.
- The quote on the previous slide clearly illustrates the importance of this view and Section 12.2 provides more illustrative evidence in support of this view.
- If profits indeed are a major determinant of innovation, one sees immediately how the market size effect plays a large role.
- Technological change can thus be considered an economic activity that responds to profit incentives.

The Value of Innovation in Partial Equilibrium

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- What is the (partial equilibrium) value of innovation and R&D to a firm in a particular industry?
- Firms in this industry can use an existing technology and the marginal cost of production is $\psi > 0$.
- Demand in this industry is given by the demand curve $Q = D(p)$, where Q denotes the quantity demanded at price p.
- The demand curve is strictly decreasing, differentiable and satisfies

$$D(\psi) > 0 \text{ and } \varepsilon_D(p) \equiv -\frac{pD'(p)}{D(p)} \in (1, \infty),$$

where the first condition ensures that demand is positive and the second that the profit-maximizing monopoly price is well-defined.

Innovation in Pure Competition I

- In this model there is a large number, N, of firms that have access to an existing technology.
- One of these firms, say number 1, also has access to a technology that leads to a process innovation, which is nonrival and nonexcludable (assume, for instance, that there is no patent system).
- The cost for this access is $\mu > 0$, and the innovation reduces the marginal cost of production to $\frac{\psi}{\lambda}$ where $\lambda > 1$.
- What incentive does this firm have to undertake the innovation?
- Since there is a large number of firms which all have access to the same technology, price will be equal to marginal cost: $p^N = \psi$ (the superscript N denotes no innovation) and the quantity demanded will equal $D(\psi) > 0$.

Innovation in Pure Competition II

- The profits of firm 1 are then: $\pi_1 = (p^N - \psi)q_1^N = 0$.
- What happens if the firm innovates?
- The innovation takes place, but since it is nonexcludable, the other firms in the industry will adopt it.
- The new equilibrium price will be $p^I = \frac{\psi}{\lambda} < \psi$ (the superscript *I* denotes innovation) and the equilibrium quantity with innovation will be $D(\frac{\psi}{\lambda}) > D(\psi)$.
- The profits of firm 1 are now $\pi_1^I = (p^I - \frac{\psi}{\lambda})q_1^I - \mu = -\mu < 0$.
- Profits are thus negative for the firm that undertakes the innovation.
- Due to the nonexcludability of the innovation the firm is not able to *appropriate* the gains of the innovation.
- Hence, under this market structure no innovation will take place.

The Social Value of Innovation

- Such an outcome is not very efficient.
- As an illustration calculate the social value of innovation, measured as the sum of consumer and producer surplus.
- If the good is always priced at marginal cost, this value is

$$
\begin{aligned}
S^I &= \int_{\frac{\psi}{\lambda}}^{\psi} D(p)dp - \mu \\
&= \int_{\frac{\psi}{\lambda}}^{\psi} [D(p) - D(\psi)]dp + D(\psi)\frac{(\lambda - 1)\psi}{\lambda} - \mu. \qquad (1)
\end{aligned}
$$

- The first term in the second line captures the increase in consumer surplus due to the lower price, the second term denotes the savings for already-produced units, and the last term is the cost of innovation.
- One sees that the value in (1) could be arbitrarily large.

Innovation in Pure Competition – Some Caveats

- The crucial problem thus is that the innovator is not able to exclude others from adopting his innovation.
- There is thus no *ex post monopoly power*.
- However, even if there is no patent system, two possibilities may provide the necessary incentives to generate innovation:
  1. trade secrecy and
  2. firm-specific innovations which cannot be used by other firms.
- Hence, there is a possibility that even under pure competition innovation takes place.

Innovation and Ex Post Monopoly I

- The model is the same as before, but assume now that if firm 1 undertakes an innovation, it can obtain a patent on it that is fully enforced.
- In this case firm 1 alone has access to a better technology – the others are stuck with the old technology.
- This monopoly power enables firm 1 to earn profits. A fact which in all likelihood encouraged the research activity in the first place.
- In analyzing the situation it makes sense to separate two cases.
- 1) *Drastic innovation*: In this case the value of λ is sufficiently high so that firm 1 becomes an effective monopolist. To determine the value of λ that leads to this situation, suppose that firm 1 indeed acts like a monopolist and chooses the price that maximizes profits, i.e.

$$\pi_1^I = D(p)(p - \frac{\psi}{\lambda}) - \mu.$$

Innovation and Ex Post Monopoly II

- Solving this problem leads to the standard formula for the monopolist's price

$$p^M \equiv \frac{\frac{\psi}{\lambda}}{1 - \varepsilon_D(p^M)^{-1}} \tag{2}$$

- An innovation is *drastic*, if $p^M \leq \psi$.
- This is the case when

$$\lambda \geq \lambda^* \equiv \frac{1}{1 - \varepsilon_D(p^M)^{-1}}.$$

- When the innovation is drastic, firm 1 sets its price equal to the monopolist's price and captures the entire market.

Innovation and Ex Post Monopoly III

- 2) *Limit pricing*: The innovation is not drastic if $p^M > \psi \Leftrightarrow \lambda^* > \lambda$.
- Firm 1 then sets the price $p_1 = \psi$ and still captures the entire market.
- If it would charge the monopolist's price, p^M, for its product the other firms cut profitably undercut it.
- Limit pricing results, for example, if some firms undertake process innovations and now have access to a better technology than their competitors.

Innovation and Ex Post Monopoly IV

Proposition In the above-described economy, suppose that firm 1 undertakes an innovation that reduces the marginal cost of production from ψ to $\frac{\psi}{\lambda}$. If $p^M \leq \psi$ (or equivalently $\lambda \geq \lambda^*$), then firm 1 sets the price $p_1 = p^M$ and makes profits

$$\hat{\pi}_1^I = D(p^M)(p^M - \frac{\psi}{\lambda}) - \mu. \tag{3}$$

If $p^M > \psi$ (or equivalently $\lambda < \lambda^*$), then it sets the limit price $p_1 = \psi$ and its profits are

$$\pi_1^I = D(\psi)\frac{(\lambda - 1)\psi}{\lambda} - \mu < \hat{\pi}_1^I. \tag{4}$$

- **Proof:** The proof is omitted.

Innovation and Ex Post Monopoly V

- The profits $\hat{\pi}_1^I$ and π_1^I also correspond to the value of innovation for firm 1, since without innovation the firm would make zero profits.
- How do these values compare to the social value of innovation which is given in (1) and what are the social values when innovation is undertaken by firm 1?
- The social surplus in the case of a drastic innovation is

$$\hat{S}_1^I = D(p^M)(p^M - \frac{\psi}{\lambda}) + \int_{p^M}^{\psi} D(p)dp - \mu$$

and the one for limit pricing is given by

$$S_1^I = D(\psi)\frac{(\lambda - 1)\psi}{\lambda} - \mu.$$

Innovation and Ex Post Monopoly VI

Proposition The following inequalities hold

$$\pi_1^I < \hat{\pi}_1^I < S^I$$

and

$$S_1^I < \hat{S}_1^I < S^I.$$

- **Proof:** The proof is omitted.
- The inequalities show that the social value of an innovation is always greater than the private value.
- The first inequality shows that since the firm will only be able to appropriate a fraction of the gain in consumer surplus due to the better technology (*appropriability effect*), a social planner is always more willing to innovate.

Innovation and Ex Post Monopoly VI

- The second inequality in the Proposition shows that whether innovation occurs or not, the gain in social surplus that could have been achieved by the social planner is always greater than in the other two cases.
- Hence, even though ex post monopoly power provides incentives to innovate, these incentives and the resulting equilibrium allocations are still inefficient.

The Replacement Effect I

- Assume that the environment is the same as before, but that firm 1 is already a monopolist under the existing technology.
- It thus sets the price

$$\hat{p}^M \equiv \frac{\psi}{1 - \varepsilon_D(p^M)^{-1}}$$

and its profits are given by

$$\hat{\pi}_1^N = D(\hat{p}^M)(\hat{p}^M - \psi). \quad (5)$$

The Replacement Effect II

- Suppose now that firm 1 undertakes an innovation and thereby reduces its marginal cost to $\frac{\psi}{\lambda}$.
- Thereafter it still remains a monopolist with profits $\hat{\pi}_1^I$ as in (3) and the monopoly price is given by p^M in (2).
- What is the value of an innovation for the monopolist?
- It is given by

$$\Delta\hat{\pi}_1^I = \hat{\pi}_1^I - \hat{\pi}_1^N = D(p^M)(p^M - \frac{\psi}{\lambda}) - D(\hat{p}^M)(\hat{p}^M - \psi) - \mu, \quad (6)$$

with $\hat{\pi}_1^I$ given by (3) and $\hat{\pi}_1^N$ given by (5).

The Replacement Effect III

Proposition It holds that $\Delta\hat{\pi}_1^I < \pi_1^I < \hat{\pi}_1^I$. This implies that a monopolist always has a lower incentive to innovate than her competitors.

- **Proof:** The proof is omitted.
- This result is often referred to as “Arrow’s replacement effect” due to Arrow’s 1962 article.
- The result shows that a monopolist would only replace her existing profits.
- A competitor on the other hand has zero profits to begin with and thus nothing to replace.
- An immediate corollary is thus that a potential entrant has stronger incentives to innovate compared to the incumbent monopolist.
- His incentive are to become the ex post monopolist and then reap the monopolist’s profits.

The Replacement Effect IV

- Entrants may then be considered the engines of process innovations.
- This observation is closely linked to the Schumpeterian growth models that will be developed in a later lecture.
- Schumpeter saw the process of economic growth as one of *creative destruction*.
- The prospect of monopoly profits drives innovation and brings about the destruction of the incumbent's rents.
- There are thus losers in the process of economic growth brought about by creative destruction.
- Political economy considerations now become important, as incumbents are possibly politically powerful and may obstruct the process of economic growth by trying to protect their monopoly rents against innovative entrants.

The Business Stealing Effect I

- By replacing the incumbent, the entrant is in effect stealing the monopolist's profits or business and this effect is accordingly called the "business stealing effect".
- However, this effect includes the possibility of excessive innovation by the newcomer.

  Proposition: The entrant may have excessive incentives to innovate, as it is possible that $\hat{S}_1^I < \hat{\pi}_1^I$.
- The intuition is that the social planner includes the monopolist's profits in his calculations as part of the producer surplus.
- In contrast the entrant only considers the profits he is going to make, if he undertakes the innovation.

The Business Stealing Effect II

- Why is the result in the proposition important?
- Whether an equilibrium involves too little or too much innovation is in general not clear.
- The answer hinges on the relative strength of the business stealing effect vs. the previously mentioned appropriability effect.

The Dixit-Stiglitz Model – Finite Number of Products I

- So far in this lecture we have conducted a partial equilibrium analysis (the household side has not been specified).
- In growth theory general equilibrium models are more interesting.
- Turning now to the Dixit-Stiglitz model (1977) which formalizes the work by Chamberlin (1933) on monopolistic competition.
- The economy is static and admits a representative household with preferences

$$U(c_1, \ldots, c_n, y) = u(C, y). \tag{7}$$

- C is a consumption index and defined as

$$C \equiv \left(\sum_{i=1}^{N} c_i^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}. \tag{8}$$

The Dixit-Stiglitz Model – Finite Number of Products II

- The sum is over N differentiated varieties of a particular good, and y denotes all other consumption.
- $u(C, y)$ is increasing, differentiable, and jointly strictly concave.
- ε denotes the elasticity of substitution between the varieties, and it is assumed that $\varepsilon > 1$.
- The identity (8) features *love-for-variety* which means that consuming a higher number of differentiated good leads to higher utility.

The Dixit-Stiglitz Model – Finite Number of Products III

- How does this work?
- As an illustration, consider the case when the varieties are consumed in identical quantities so that

$$c_1 = \ldots = c_N = \frac{\bar{C}}{N}$$

- Substituting this expression into (7) and (8) leads to

$$U\left(\frac{\bar{C}}{N}, \ldots, \frac{\bar{C}}{N}, y\right) = u\left(N^{\frac{1}{\varepsilon-1}}\bar{C}, y\right).$$

- This expression is strictly increasing in N and it implies that utility is higher the larger the number of varieties for a given total of $\bar{C}$.

The Dixit-Stiglitz Model – Finite Number of Products IV

- The individual budget constraint is given by

$$\sum_{i=1}^{N} p_i c_i + y \leq m \tag{9}$$

where the price of y has been normalized to 1, the price of variety i is given by p_i and total household income in terms of good y is denoted by m.

- Consumer maximization then implies the following first-order conditions:

$$\left(\frac{c_i}{c_{i'}}\right)^{-\frac{1}{\varepsilon}} = \frac{p_i}{p_{i'}} \quad \text{for any} \quad i, i'.$$

The Dixit-Stiglitz Model – Finite Number of Products V

- As mentioned C denotes the consumption index – what is then the corresponding price index?
- This is called the *ideal price index* and denoted P. It is defined via the first-order conditions for the consumption index

$$\left(\frac{c_i}{C}\right)^{-\frac{1}{\varepsilon}} = \frac{p_i}{P} \quad \text{for any} \quad i = 1, \ldots, N \tag{10}$$

from which the ideal price index

$$P \equiv \left(\sum_{i=1}^{N} p_i^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}} \tag{11}$$

can be derived.

The Dixit-Stiglitz Model – Finite Number of Products VI

- Combining equations (10) and (11) with the budget constraint in (9) leads to a budget constraint expressed in terms of C:

$$PC + y \leq m. \tag{12}$$

- Maximization of the utility function in (7) subject to the restriction above yields

$$\frac{\frac{\partial u(C,y)}{\partial y}}{\frac{\partial u(C,y)}{\partial C}} = \frac{1}{P}.$$

- By using the budget constraint and the strict joint concavity of the utility function this condition can be written as

$$y = g(P, m) \quad \text{and} \quad C = \frac{m - g(P, m)}{P} \tag{13}$$

for some function $g(P, m)$.

The Dixit-Stiglitz Model – Finite Number of Products VII

- What about the production side?
- Assuming that each variety is produced by a single producer, this producer is in effect a monopolist for his particular variety and faces the following profit maximization problem:

$$\max_{p_i \geq 0} \left(\underbrace{\left(\frac{p_i}{P}\right)^{-\varepsilon} C}_{=c_i} \right)(p_i - \psi).$$

- Solving this problem is not straightforward, as P and C are possibly functions of p_i.
- If however the number of varieties is sufficiently large, this effect may be ignored and it can be shown that the profit-maximizing price is given by

$$p_i = p = \frac{\varepsilon}{\varepsilon - 1}\psi \quad \text{for each} \quad i = 1, \ldots, N. \tag{14}$$

The Dixit-Stiglitz Model – Finite Number of Products VIII

- Hence each firm charges the same price and the ideal price index is thus
$$P = N^{-\frac{1}{\varepsilon-1}} \frac{\varepsilon}{\varepsilon-1}\psi. \tag{15}$$
- With this information at hand one can obtain each firm's profits:
$$\pi_i = \pi = N^{-\frac{\varepsilon}{\varepsilon-1}} C \frac{1}{\varepsilon-1}\psi \quad \text{for each} \quad i = 1, \dots, N.$$
- It holds that $\frac{\partial \pi}{\partial \varepsilon} < 0$, $\frac{\partial \pi}{\partial C} > 0$ and $\frac{\partial \pi}{\partial N} < 0$.

The Dixit-Stiglitz Model – Finite Number of Products IX

- The last effect captures that for a given C the higher the number of varieties the less is spend on each.
- Nonetheless, the total effect of N on profits may be positive.
- This can be seen by substituting the ideal price index in (15) into (13) which leads to

$$C = N^{\frac{1}{\varepsilon-1}} \frac{\varepsilon - 1}{\varepsilon\psi} \left(m - g \left(N^{-\frac{1}{\varepsilon-1}} \frac{\varepsilon}{\varepsilon - 1} \psi, m \right) \right),$$

and

$$\pi = \frac{1}{\varepsilon N} \left(m - g \left(N^{-\frac{1}{\varepsilon-1}} \frac{\varepsilon}{\varepsilon - 1} \psi, m \right) \right).$$

The Dixit-Stiglitz Model – Finite Number of Products X

- The specific form of the function $g(\cdot)$ (or ultimately the form of the utility function) determines if profits are increasing in N.
- This is not an immediately intuitive result – one would expect that a greater number of competitors would reduce an individual firm's profits.
- However, the crucial aspect is the love-for-variety effect that is represented in the Dixit-Stiglitz preferences.
- This effect is responsible for an increase in demand and is often called an “aggregate demand externality”.
- Due to the love-for-variety effect, a higher N increases the utility that the individuum receives from consuming each variety.

The Dixit-Stiglitz Model – Finite Number of Products XI

- Why is it an externality?
- The effect works via the ideal price level P, since a higher N reduces P which then leads to a higher C.
- It therefore corresponds to a pecuniary externality.
- Note that this externality possibly has first-order welfare effects as well (the First Welfare Theorem can no longer be applied).

The Dixit-Stiglitz Model – Continuum of Products I

- Now there is a continuum of products; otherwise the model is similar to the one just discussed.
- Specifying the model in this way makes it more tractable and in addition the profit-maximizing price in (14) is no longer an approximation.
- The utility function of the household then changes to

$$U\left([c_i]_{i\in[0,N]}, y\right) = u\left(C, y\right)$$

with C given by

$$C \equiv \left(\int_0^N c_i^{\frac{\varepsilon-1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}}.$$

The Dixit-Stiglitz Model – Continuum of Products II

- The budget constraint in this version of the model is

$$\int_0^N p_i c_i di + y \leq m.$$

- Going through similar steps as in the model with a discrete number of product varieties leads again to the first-order conditions in (10)
- The appropriate ideal price index is

$$P = \left(\int_0^N p_i^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}},$$

and equations (12) and (13) hold as well.

The Dixit-Stiglitz Model – Continuum of Products III

- Since each firm in this model is infinitesimal, it cannot influence P and C via its price.
- This means that the pricing decision in (14) applies exactly and profits are again given by

$$\pi = \frac{1}{\varepsilon N}\left(m - g\left(N^{-\frac{1}{\varepsilon-1}}\frac{\varepsilon}{\varepsilon-1}\psi, m\right)\right).$$

- By means of the above expression it is possible to endogenize the entry margin. If N is infinite and a given firm can pay a fixed cost $\mu > 0$ to adopt a variety and enter the market, then the following free-entry condition that dates back to Chamberlin (1933) has to hold

$$\frac{1}{\varepsilon N}\left(m - g\left(N^{-\frac{1}{\varepsilon-1}}\frac{\varepsilon}{\varepsilon-1}\psi, m\right)\right) = \mu.$$

The Dixit-Stiglitz Model – Too little or too much entry?

- Is there too little or too much entry?
- The aggregate demand externality would suggest that there is too little entry, as the entering firms do not take into account the positive effect they have on other firms.
- On the other hand, the business stealing effect discussed previously continues to be present. So there is a chance that entry by new competitors reduces the demand for existing goods.
- In the end the question of whether entry is optimal in these types of models hinges, for instance, on the specific parameter values in the model.

The Dixit-Stiglitz Model – Limitations

- In the Dixit-Stiglizt model the markup of each firm is independent of the available number of varieties.
- This feature makes the model tractable, but it is not very realistic.
- In most industrial organization models this markup over marginal cost declines when the number of available products increases.
- However, incorporating this effect would make endogenous growth models less tractable.
- Also, sustainable growth would not be possible if the markup would decline as N increases, since innovation would stop in this case.
- The problem may be circumvented, but these alternative setups are harder to analyze and hence the focus on the Dixit-Stiglitz model in the literature.

Conclusion

- The importance of ex post monopoly power for the creation of incentives for innovation has been highlighted.
- Three effects: the replacement, the appropriability, and the business stealing effect have been discussed.
- The first implies that entrants have a stronger incentive to innovate, as they potentially replace zero profits with the monopolist's profits, the second effect captures the fact that the private value of innovation is often less than its social value, and the last effect implies that entrants strive to innovate and "steal" the incumbent's monopoly rents.
- Since the latter two effects work in opposite directions the market structure and the parameters of the model decide whether the equilibrium incorporates too little or too much innovation.
- The Dixit-Stiglitz model was introduced which formalizes Chamberlin's discussion of monopolistic competition.
- Very tractable framework, since the markup is independent of the number of competitors.