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Productive public input, integration and agglomeration

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ABSTRACT

This paper analyzes, within a regional growth model, the impact of a productive governmental input and integration on the spatial distribution of economic activity. In doing so, two benchmark cases (i) equal distribution, and (ii) agglomeration in the sense of a core–periphery structure as well as the corresponding transition processes are discussed. Integration is understood as enhancing inter-territorial cooperation and it describes the extent to which one region may benefit from the other region's public input. Both integration and the characteristics of the public input crucially affect whether or not agglomeration arises and hence to which extent economic activity is concentrated. Key results are: Intensifying integration reduces the strengths of agglomeration forces and the corresponding degree of concentration will be lower. Relative congestion leads to an overestimation of capital returns and capital growth in the core region is faster due to congestion thereby leading to suboptimally high concentration.

1. Introduction

Supporting convergence as well as intensifying European territorial cooperation is among the key objectives of European regional policy for the period of 2007–2013. One of the instruments to reach these goals is the further improvement of the transport infrastructure, which is mostly financed by resources of structural and cohesion funds. Considering whether much and/or better infrastructure is apt to reach the goal of convergence is part of both theoretical and empirical analysis. This work is based (Aschauer, 1989a, b), who highlights the economic impact of infrastructure as it enhances productivity of the private inputs. Accordingly, infrastructure investment in lagging regions is justified as it fosters catching-up processes that on top of everything are the faster the more lagging the considered country or region is. However, recent contributions in the macroeconomic literature find more modest returns to infrastructure investment (see e. g., Gramlich (1994) or more recently de Haan and Romp (2007) for a survey on the empirical literature). In any case, the impact of productive government expenditure as a growth determinant is undoubted.

The growth impact of governmental activity lies at the heart of Barro-type endogenous growth models that have been continuously refined to allow for various characteristics, among them congestion, of the public input (see e. g., Barro (1990) for the seminal work and Glomm and Ravikumar (1994a, b) or Turnovsky (2000) for an overview). However, all these considerations focus on the productive impact of a public input within a single country and, if they analyze convergence at all, they view it as a process leading to an equilibrium growth path of the considered country. In any case, this perspective of an isolated economy is not apt to analyze issues of convergence as intended by the EU, as it may not explain the spatial distribution of economic activity and how it is affected by governmental policy.

At the same time it is well recognized that since the era of the industrial revolution, growth and agglomeration (i.e., spatial concentration of economic activity) are mutually self-reinforcing processes (see e. g., Martin and Ottaviano, (2001)). This perspective picks up the concerns lying at the heart of the so-called 'new economic geography' (see Krugman (1991) or Brakman et al. (2009) for an excellent overview; or Duranton and Puga (2004) for an overview on the underlying micro-foundations of agglomeration economies). Corresponding models single out imperfect competition, increasing returns, and transportation costs as fundamental resources shaping the economic landscape, but few focus on governmental activity. Exceptions are the works of Martin and Rogers (1995), who focus on the role of infrastructure as facilitating transactions, i.e., the trade within and between countries or Brakman et al. (2002, 2008) who also show that governments may affect the economic landscape through the provision of public inputs. Aside from the broadly discussed 'race-to-the-bottom' result in the context of tax-financing schemes, the authors also focus on the effect of governments that provide public inputs to enhance productivity of private inputs. Puga (2002) analyzes the impact of regional policy expenditures on mitigating regional disparities and highlights that an undifferentiated consideration of infrastructure

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neglects that different characteristics — for instance, the prevailing degree of congestion — also operate differently in a spatial context.

However, though the formerly mentioned new economic geography models include regional governmental policies, they mostly consider infrastructure in reference to reduced transportation costs or in the context of the resulting fiscal effects; by contrast, most Barro-type growth models assume a productive governmental input but neglect regional interaction. An exception is the work of Alonso-Carrera et al. (2009), who analyze within a general equilibrium model how public investment causes imbalances in regional development and how fiscal policy may be used in order to overcome these spatial disparities. In doing so, Alonso-Carrera et al. (2009) simulate how public investment in one region affects economic performance of other regions thereby taking into account the opportunity cost of public investment if the latter is not implemented at the most efficient location. The regions are linked by inter-regional spillovers but the issue of integration is not directly addressed.

These shortcomings of the existing literature are the starting point of the present model: We analyze the impact of regional policy on agglomeration. Regional policy thereby includes the provision of public capital that basically may be interpreted in a broad sense as comprising any facility, good or institution provided by the government that enhances the productivity of the other private inputs. This broad view of the public input allows for a consideration of physical infrastructure such as roads, airports, and telecommunication networks, but also as basic research or training networks of education infrastructure. This broader interpretation of the public input as well as the corresponding productive impact has already been incorporated in the literature, e.g., by Garcia-Milà and McGuire (1992), who explicitly differentiate public inputs in physical infrastructure and education. These different types formally may be represented by integrating a congestion function adopted from Eicher and Turnovsky (2000), which includes congestion as well as capital spillovers. We extend their modelling of the governmental input and implement it in a modified version of the regional growth model of Bröcker (2003), who for his part focuses on learning-by-doing and inter-regional knowledge diffusion.

In addition, we focus on integration between the two regions as the extent to which one region may benefit from the other region's public input. With this formulation we rely on Alesina and Spolaore (2003, Chapter 6) and are broader than the usual approach of the new economic geography, which assumes that integration predominantly strengthens agglomeration as it reduces transport costs. Our setting is in line with the goal of the European regional policy mentioned before, namely of enhancing European territorial cooperation. It will be shown that integration, in contrast to the formulation of the European Union, is not exclusively a goal on its own but might even serve as an instrument to reach the goal of convergence. For a differentiated public input, integration may either be interpreted as better connectivity of infrastructure networks, e. g., via bridges. Or, in case of public activity as education, one might interpret integration as the degree to which foreign degrees are accepted at the regional labor market. Integration may then also be achieved by increasing the flow of ideas between regions as already argued by Rivera-Batiz and Romer (1991) and others. We assume identical production technologies with constant returns to the private inputs labor and capital for the two regions. Convergence in this setting is interpreted as development towards equal distribution of economic activity and hence equalized standards of living. Labor is immobile while capital accumulation is taking place in the region with the higher productivity. The resulting equilibrium is based on equalized productivities of capital, and it determines the equilibrium distribution of capital and hence production and welfare. Equal distribution would then represent a situation of convergence. Put differently, a core–periphery structure would contradict the goal of equalizing the standard of living among the regions.

Depending upon the interaction between agglomeration and dispersion forces, multiple equilibria with different stability characteristics may arise. As in the work of Brakman et al. (2008), the spreading equilibrium is unstable. All things being equal, the resulting equilibrium depends upon the initial distribution of capital and the regional endowments with immobile labor. It is shown that the bifurcation point is a function of congestion, capital spillovers and integration. The endowment with immobile labor acts as the threshold value that determines the final equilibrium capital distribution. Agglomerations reflect equilibrium capital distributions with different regional capital stocks. In analogy to Krugman (1991), the region displaying the bigger capital stock may then be interpreted as being the core, while the other region is the periphery.

As argued before, in the light of this model, convergence in the sense of the European Union may be interpreted as a reduction in concentration. Basically, this may be derived either by integration and/or by the type of the governmental input provided as determined by the degrees of congestion and spillovers. Integration reduces concentration, since it allows the periphery to access the core's public input and hence to benefit from its productivity. In contrast, relative congestion is associated with a negative capital externality and aggravates concentration. Individuals know that government spendings in the core are higher than in the periphery. Nevertheless, they overlook the fact that aggregate capital will rise due to capital accumulation. Hence, in the presence of relative congestion the available amount of the public input will be reduced due to capital accumulation. As a consequence, the decentralized equilibrium ends up in suboptimally high concentration, which is due to the externalities and reinforces regional disparities. The impact of capital spillovers may be ambiguous: Basically, agglomeration forces are strengthened by capital spillovers, since the productivity advantage of the core gains importance. Nevertheless, strong spillovers may smoothen concentration if combined with a high degree of relative congestion. This is the consequence of decreasing marginal returns in the governmental input. In any case, integration lowers concentration forces unequivocally. Thus the argument of this paper directly may be assigned to the argument of Diego Puga in the 2009 World Development Report, where he describes the recent economic landscape and the pursued policy goals as concentration is the rule, convergence is the objective, and integration is the answer (see World Bank (2009, p. 41)). Our paper confirms this view and the policy recommendation includes the statement that, in order to achieve the goal of convergence, the European Union should more focus on integration which according to our arguing might not only serve as goal but may even be used as an instrument to achieve convergence.

The remainder of the paper is a follows: After presenting the analytical framework in the next section, balanced steady states are derived in Section 3. Section 4 explores the determinants of agglomeration, while Section 5 carries out numerical simulations. Efficiency arguments are discussed in Section 6. Section 7 concludes, while formal derivations are relegated to the Mathematical appendix.

2. The analytical framework

2.1. Firms

Firms in both regions $\iota = 1, 2$ produce the homogenous good, Y_ι, by the same Cobb–Douglas technology. The inputs used in each region are immobile labor,[1] L_ι, and private capital, K_ι. Furthermore, output depends upon regional access to a global public input that is measured

[1] We exclude mobile labor from the analysis, because it essentially follows the distribution of capital.

by an index, $D_\imath$. The production function for a representative firm in region $\imath$ is given by

$$Y_\imath = L_\imath^\lambda K_\imath^\alpha D_\imath^\gamma, \quad 0<\lambda,\alpha,\gamma<1 \tag{1}$$

The global public input, $D_\imath$, includes the regional public inputs, $G_{s\imath}$, that are separately provided by both regions. The firm's access to the other region's public input may be limited as parameterized by $0\leq\beta\leq 1$ and we assume

$$D_1 = G_{s1} + \beta G_{s2} \tag{2a}$$

$$D_2 = G_{s2} + \beta G_{s1} \tag{2b}$$

Correspondingly, the parameter β may be interpreted as a measure for the extent of integration between the two regions. If $\beta=0$, firms in each region only benefit from the public input provided by their local governments; consequently, the scope of governmental policy is restricted to their own region. In contrast to this, $\beta>0$ implies that firms in one region also have (at least partial) access to the other region's public input. What we have in mind is the following: If the government of a certain region provides education for the early childhood, with the goal to increase the productivity in its own region, the impact on the other region's productivity probably will not be affected significantly (at least if labor is immobile). Formally, β will be close to zero. The same argument applies to the provision of a university that restricts the access to students stemming from its own region. If, in contrast to this, the government of region 1 provides universities that are open to students from region 2 (and if graduates return to their home region), productivity in both regions will increase as a consequence of governmental activity in one single region. Then, β will be positive. Another example could also be given by the provision of a public infrastructure. Consider two countries that both provide a road network as public input. As long as these networks are not connected, the spatial scope of governmental policy is restricted to its own region. Firms in region $\imath$ only benefit from their own region's roads, $\beta=0$. But if now, e.g., ferries, connecting roads, tunnels or bridges are established, the road network in region 1 may be also used by firms of region 2. Formally, β increases up to $\beta=1$; this reflects the other polar case in which firms in both regions have access to the entire public inputs provided in both regions. Then the global public input covers both road networks and the two regions are perfectly integrated.[2] Another example can be given by the validity area of patents that describe another facet of the spatial scope of governmental activity.

The modelling of the governmental input is adopted from Eicher and Turnovsky (2000), and the public input provided by the local government in region $\imath$ may be characterized as follows

$$G_{Sl} = G_l\left(\frac{K_l}{\overline{K}_l}\right)^{\varepsilon_R}\overline{K}_l^{\varepsilon_A}, \quad 0\leq\varepsilon_R\leq 1, \quad -\alpha\leq\varepsilon_A\leq 1 \tag{3}$$

whereby $\overline{K}_l$ denotes the aggregate stock of private capital in region $\imath$, and G_l denotes the aggregate flow of government expenditure. Function (4) incorporates the potential for the public good to be associated with alternative degrees of scale effects, denoted by ε_A, or congestion, denoted by ε_R. In contrast to Eicher and Turnovsky (2000), we do not restrict the sign of ε_A to be negative.[3] Instead, we will analyze the centripetal force of positive spillovers as well as the centrifugal force of congestion and focus on the interdependent impact of both forces on equilibrium distribution of economic activity. In particular, we will demonstrate in Sections 4 and 5 that in the dynamic setting we analyze, relative congestion facilitates the appearance of agglomeration in the sense that it makes equal distribution of capital less probable. Since individuals neglect their influence on aggregate capital, there is a negative externality in capital accumulation and capital return is overestimated by the firms. Hence, a higher degree of relative congestion will support the agglomeration force induced by scale effects.

Altogether, the public services can be classified into the following categories.

(i) If $\varepsilon_A=\varepsilon_R=0$, government services constitute a pure public good in the sense of Samuelson (1954) and $G_{s\imath}=G_\imath$. The public input is available equally to each individual within region $\imath$, independent of the usage of others.[4] Governmentally provided basic research may serve as an example. Its usage by one firm does not affect the possible usages of the others. The same is true for the usage of the public input by firms from other regions.

(ii) Relative congestion arises if $\varepsilon_R>0$: This reflects situations in which the level of the public input available to the individual is tied to this individual's usage of capital. As already explained, $\varepsilon_R=0$ corresponds to a nonrival pure public input, while $\varepsilon_R=1$ reflects a situation of proportional relative congestion. Accordingly, the cases $0<\varepsilon_R<1$ correspond to situations of partial relative congestion, in the sense that given the individual stock of capital, government spendings can increase at a slower rate than does $K_\imath$ and still provide a fixed level of services to the firm. An example for $\varepsilon_R\leq 1$ could be the provision of a road network. In extreme, it is proportionally congested and each of the $N_\imath$ firms within region $\imath$ may use $1/N_\imath$ parts of the entire public input, $G_\imath$, for production.[5] Relative congestion reflects the disadvantages of concentration. For a given amount of governmental input (e. g., infrastructure), the individually available amount is smaller, the more individuals make use of it, or – put differently – the larger the aggregate capital stock. A single-lane highway is more productive for the individual firm, the less other trucks (aggregate capital) use it.

(iii) Intra-regional spillovers given that $\varepsilon_A>-1$: In any dynamic equilibrium, aggregate capital and governmental expenditures grow at the same constant rate, as will be demonstrated in the context of (26). Hence, with $\varepsilon_A>-1$, positive effects of capital accumulation arise, and individuals benefit from the accumulation of the others. This externality can be interpreted as a net externality, or in the sense of Romer (1986); an example could be the outcomes of research centers that are financed by non-distortionary taxes.[6] The positive effects of the governmental input increase with the absolute size of the economy. Learning by doing is promoted by governmentally provided schools and universities, and the productivity increase induced by schools and universities is enhanced by a high degree of automation displayed by high capital intensity.

For the production technology (1) to allow for endogenous growth in both regions, an additional constraint has to be imposed, namely

[2] Note that both limiting cases, $\beta=0$ and $\beta=1$, characterize an extreme and unrealistic world but may be well useful as benchmark cases.

[3] Nevertheless, in order to allow for ongoing growth, $-\alpha\leq\varepsilon_A$ has to be satisfied, as will be explained below.

[4] Since only few examples of such pure public goods exist, this case should be treated primarily as a benchmark.

[5] As Turnovsky (1996, p. 364) argues, the case $\varepsilon_R>1$ describes a situation where congestion is so great that the public input must grow faster than the economy in order for the level of services provided to the individual firm to remain constant. This case is unlikely at the aggregate level, but may well be plausible for local public goods (see also (Edwards (1990)). A local public good could be a harbor that is provided by the regional government. Nevertheless, it also may be used by individuals coming from outside the region. However, Turnovsky, (1996) argues in the context of a one-country model; hence, it is not possible to apply the argumentation carried out there 1:1 to our framework. Here, possible utilization of an input that is provided by the other region is parameterized by $\beta>0$ and not by $\varepsilon_R>1$.

[6] Note that the positive spillovers in the model of Romer (1986) do not exactly correspond to the framework of this model, since there the spillovers arise independent of governmental activity.

$\alpha+\gamma(1+\varepsilon_A)=1$. This ensures constant returns to private capital, the accumulable factor.[7]

From Eqs. (1), (2a), (2b) and (3), the output of the individual (representative) firm in region 1 is given by

$$Y_1 = L_1^{\lambda}K_1^{\alpha}(G_1K_1^{\varepsilon_R}\overline{K}_1^{\varepsilon_A-\varepsilon_R} + \beta G_2K_2^{\varepsilon_R}\overline{K}_2^{\varepsilon_A-\varepsilon_R})^{\gamma} \tag{4}$$

and output of the representative firm in region 2 may be derived accordingly. If $\beta=0$, the scale elasticity of Y_ι is given by $\lambda+\alpha+\gamma(1+\varepsilon_A)$. Hence, for all feasible levels of ε_A, production is characterized by increasing returns to the local inputs and this is reinforced with increasing ε_A. The private (average) capital productivities in both regions evolve according to

$$\frac{Y_1}{K_1} = L_1^{\lambda}\left(1+\frac{\beta}{g_s}\right)^{\gamma}\left(\frac{G_1}{K_1}\right)^{\gamma}N_1^{\gamma(\varepsilon_A-\varepsilon_R)} \tag{5a}$$

$$\frac{Y_2}{K_2} = L_2^{\lambda}(g_s+\beta)^{\gamma}\left(\frac{G_2}{K_2}\right)^{\gamma}N_2^{\gamma(\varepsilon_A-\varepsilon_R)} \tag{5b}$$

whereby the following variables are utilized

$$g \equiv G_1/G_2, \quad g_s \equiv G_{s1}/G_{s2} = g k^{\varepsilon_R}\overline{k}^{\varepsilon_A-\varepsilon_R} \quad \text{with} \quad k \equiv K_1/K_2, \quad \overline{k} = \overline{K}_1/\overline{K}_2 \tag{6}$$

Average productivities thus depend on the distribution of capital and governmental activity across regions, as incorporated within g_s, the ratio G_ι/K_ι, as well as on the number of firms located in each region, N_ι, and on the type of the public input, as incorporated within the term $N_\iota^{\gamma(\varepsilon_A-\varepsilon_R)}$.

2.2. Households and regional growth

The infinitely lived households possess identical isoelastic preferences, and maximize lifetime utility out of consumption,

$$U_l = \int_0^{\infty}\frac{\sigma}{\sigma-1}C_l(t)^{\frac{\sigma-1}{\sigma}}e^{-\rho t}dt \quad \rho>0, \quad 0<\sigma<1 \tag{7}$$

The subjective discount rate is denoted by ρ, σ is the elasticity of intertemporal substitution, and $C_\iota(t)$ describes consumption in region ι.[8]

Households save by accumulating a risk free asset. The asset value equals the value of the stock of capital at any point in time; hence, the asset value in region ι at time t equals $V\iota(t)\equiv q_1(t)K_{1\iota}(t)+q_2(t)K_{2\iota}(t)$, where q_ι denotes the stock price of capital installed in region ι. The immobile workers earn labor income as well as capital income from investment in both regions. Wages in region ι are denoted by $w_\iota(t)$. The total income in region ι evolves according to

$$\dot{V}_\iota(t) = w_\iota(t)L_\iota(t) + (r(t)-\delta)V_\iota(t)-C_\iota(t)-T_\iota(t) \tag{8}$$

with $r(t)$ denoting the interest rate determined in capital market equilibrium, δ as the constant depreciation rate of private capital and $T_\iota(t)$ a lump-sum tax that is used to finance the provision of the public input. To fully describe the optimization problem, the transversality conditions

$$\lim_{t\to\infty} K_{1\iota}(t)\xi_\iota(t) = 0 \qquad \lim_{t\to\infty} K_{2\iota}(t)\xi_\iota(t) = 0 \tag{9}$$

have to be met, where ξ_ι denotes the shadow value of capital in region ι. Maximizing Eq. (7) subject to the accumulation constraint (8) leads to the Hamiltonian

$$\mathcal{H}_\iota = \frac{\sigma}{\sigma-1}C_\iota(t)^{\frac{\sigma-1}{\sigma}}e^{-\rho t} + \xi_\iota(w_\iota(t)L_\iota(t) + (r(t)-\delta)V_\iota(t)-C_\iota(t)-T_\iota(t)) \tag{10}$$

with optimal consumption described by the necessary conditions

$$\frac{\partial\mathcal{H}_\iota}{\partial C_\iota} = C_\iota^{-\frac{1}{\sigma}}e^{-\rho t}-\xi_\iota \overset{!}{=} 0 \tag{11a}$$

$$\frac{\partial\mathcal{H}_\iota}{\partial V_\iota} = \xi_\iota(r(t)-\delta) \overset{!}{=} -\dot{\xi}_\iota \tag{11b}$$

and leading to the well known growth rate of consumption as[9]

$$\frac{\dot{C}_\iota}{C_\iota} = \sigma(r-\delta-\rho) \equiv \varphi \tag{12}$$

Households in both regions realize identical consumption growth, a direct consequence from homothetic preferences together with equal investment opportunities. Moreover, due to constant average returns of capital (see Eqs. (5a) and (5b)), the consumption–wealth ratio is constant and hence the growth rates of consumption, capital and income coincide. An increase in capital return, r, will increase the growth rate due to strengthened incentives for capital accumulation. In contrast, an income tax would reduce net capital return and therefore decrease the growth rate. It is well known from growth literature that a lump-sum tax $T_\iota(t)$ is growth neutral, since it does not influence capital return. As we focus on the agglomeration effects of productive governmental expenditures, we restrict governmental revenues to the growth neutral lump-sum tax. Nevertheless, the results for the case of income taxation can be obtained by redefining r to be the net capital return.

3. Balanced steady states

The equilibrium is based on equalized productivities of private capital. Individuals in the two regions are able to hold capital in region 1 or in region 2. Physical capital is only mobile as long as it is not yet nailed down. Hence, the adjustment process of marginal capital returns takes time. As capital is immobile once it has been installed, it may not be relocated to the other region. Therefore, net investment in either region is nonnegative and given by

$$I_\iota = \dot{K}_\iota - \delta K_\iota \geq 0 \tag{13}$$

With q_ι denoting the stock price of capital installed in region ι, the following conditions are complementary and must be fulfilled for sustained investment in region ι

$$I_\iota \geq 0, \quad q_\iota \leq 1, \quad I_\iota(1-q_\iota) = 0 \tag{14}$$

[7] This interdependence between the parameters implies an adjustment of the values of α or γ whenever a change in absolute congestion, ε_A, is analyzed. Expressed as $\gamma=(1-\alpha)/(1+\varepsilon_A)$ and combined with $0<\gamma\leq 1$, this constraint results in $-\alpha\leq\varepsilon_A$, which has to be imposed to enable ongoing growth. Otherwise capital productivity would not suffice to promote endogenous growth.

[8] As the households' preferences are homothetic, we prefer to analyze the optimization problem of the collectivity of the households in order to avoid too many indices. Hence, C_ι is aggregate consumption in region ι. Nevertheless, optimization of individual utility of each household would yield the same results.

[9] In what follows time indices will be suppressed.

No-arbitrage applies if capital in both regions yields identical rates of private return

$$(r+\delta)q_\iota = \dot{q}_\iota + \frac{\partial Y_\iota}{\partial K_\iota} \tag{15}$$

Since we abstract from adjustment costs, the marginal costs for installing an additional unit of capital in region ι is unity. Consequently, marginal costs and marginal returns of an additional unit of capital are equalized if $q_\iota = 1$, and as long as $q_\iota = 1$, private investors are willing to invest in region ι.[10] Then $\dot{q}_\iota = 0$; and according to (15), the interest rate equals the net marginal product of capital, $r = \partial Y_\iota / \partial K_\iota - \delta$ and investment is positive, $I_\iota > 0$. If instead $q_\iota < 1$, no investment will take place. Then $I_\iota = 0$. Since individuals only invest in the region with the higher capital return, positive investment in both regions is only feasible if marginal capital productivities coincide. Then both capital stocks grow at the same rate and the capital ratio, k, is constant.

Denote the ratio of marginal capital productivities with

$$R \equiv \frac{\partial Y_1 / \partial K_1}{\partial Y_2 / \partial K_2} \tag{16}$$

A balanced steady state is characterized by a stationary capital distribution, i.e., by $R = 1$. Then ongoing positive investment in both regions arises and capital stocks in both regions grow according to Eq. (12), with r being derived from Eq. (4). Note that an income tax would not affect the balanced steady state as long as the tax rates in both regions are equal. Hence, the results about steady state agglomeration are independent from the assumptions about government revenues. Our restriction of government revenues to the growth neutral lump-sum tax is only for notational convenience and without loss of generality.

In case of initial productivity disparities (i.e., $R \neq 1$), the prevailing capital ratio is not stationary; however, over time, transitions to a steady state with k increasing (if $R > 1$) or decreasing (if $R < 1$) will take place. Hence an equilibrium is only attained after a certain transition period, but k converges to a stable equilibrium in finite time. Since we assumed that capital is immobile once it has been nailed down, a transition with increasing k implies that during the transition period there is only investment in region 1 and no investment in region 2. The capital stock in region 2 then declines with the depreciation rate, δ.

Assume that initially capital in region 1 is more productive. Then the transition may be described by the following differential equations

$$\dot{K}_1 = Y_1 + Y_2 - \delta K_1 - C - (G_1 + G_2) \tag{17a}$$

$$\dot{K}_2 = -\delta K_2 \tag{17b}$$

$$\frac{\dot{C}}{C} = \sigma\left(\frac{\partial Y_1}{\partial K_1} - \delta - \rho\right) \tag{17c}$$

$$\dot{q}_2 = \frac{\partial Y_1}{\partial K_1} q_2 - \frac{\partial Y_2}{\partial K_2} \tag{17d}$$

which hold as long as $q_2 < 1$. Eq. (17a) is the goods market equilibrium condition; Eq. (17b) is due to exclusive investment in region 1; Eq. (17c) describes the Keynes–Ramsey rule; and Eq. (17d) is the equilibrium condition of the asset market. Note that in Eq. (17a) it is assumed that the provision of $G_1 + G_2$ is realized out of global income $Y_1 + Y_2$.[11]

[10] If $q_\iota > 1$, investment would be infinite; hence to analyze balanced steady states and the corresponding transitions, it is sufficient to deal with $q_\iota = 1$.

[11] The regional decision about the governmental input is described in Section 6.

4. Determinants of agglomeration: core and periphery

4.1. Equilibrium and government expenditure

In the previous section we demonstrated that a steady state capital distribution is characterized by equal marginal productivities of the capital stocks in both regions, hence $R = 1$. As long as marginal productivities of capital differ, capital will be invested exclusively in the region with the higher marginal product. To study the model's dynamics, one has to analyze how productivities of private capital in both regions depend on the regional distribution of capital as well as on governmental activity. To do so, the ratio R may be derived from the specified production function (4), together with Eq. (18).

Utilizing Eq. (4) and denoting $l \equiv L_1/L_2$, the output ratio of both regions can be written as

$$\frac{Y_1}{Y_2} = l^\lambda k^\alpha \left(\frac{g_s + \beta}{1 + \beta g_s}\right)^\gamma \tag{18}$$

Lower case letters reflect the distribution of the respective variable across the two regions as given by Eq. (6). For given production elasticities and given l, relative output of the regions only depends on the distribution of private capital, k, as well as on governmental activity, g_s. The latter also includes the spatial scope via spillovers, ε_A, the congestion parameter, ε_R, and the extent of inter-regional integration as measured by β.

Note that since we focus on a growing economy, we assume that the public input grows with the equilibrium growth rate. Governments in both regions set the aggregate expenditure levels, G_ι, as a constant fraction, Θ_ι, of aggregate capital, $\bar{K}_\iota$, namely[12]

$$G_\iota = \Theta_\iota \bar{K}_\iota, \quad 0 < \Theta_\iota < 1 \tag{19}$$

An expansion in government expenditure is then parameterized by an increase in the capital share, $\boldsymbol{\Theta}_\iota$. Additionally we have to take into account that in equilibrium $\bar{K}_\iota = N_\iota K_\iota$ applies. Then

$$\tilde{g}_s = \theta k^{1+\varepsilon_A} n^{1+\varepsilon_A-\varepsilon_R} \tag{20}$$

defines the equilibrium ratio of governmental activity, with $\theta \equiv \boldsymbol{\Theta}_1 / \boldsymbol{\Theta}_2$ and $n \equiv N_1/N_2$.

In equilibrium the ratio of marginal capital productivities turns out to equal

$$R = l^\lambda k^{\alpha-1} \left(\frac{\tilde{g}_s + \beta}{1 + \beta \tilde{g}_s}\right)^{\gamma-1} \cdot \left(\frac{\alpha(\tilde{g}_s + \beta) + \gamma\varepsilon_R \tilde{g}_s}{\alpha(1 + \beta\tilde{g}_s) + \gamma\varepsilon_R}\right) \tag{21}$$

Taking logarithms, after some simple manipulations, yields

$$R \gtrless 1 \quad \Leftrightarrow \quad i(k) \gtrless -\lambda \ln l \tag{22}$$

with

$$i(k) \equiv (\alpha - 1)\ln k + (\gamma - 1)\ln\left(\frac{\tilde{g}_s + \beta}{1 + \beta\tilde{g}_s}\right) + \ln\left(\frac{\alpha(\tilde{g}_s + \beta) + \gamma\varepsilon_R \tilde{g}_s}{\alpha(1 + \beta\tilde{g}_s) + \gamma\varepsilon_R}\right) \tag{23}$$

Referring to the equilibrium concept, balanced steady states are attained at those capital ratios, k^*, that solve $i(k^*) = -\lambda \ln l$. Then $R = 1$ and the marginal capital productivities are equalized across the regions. Since both regions then grow at constant rates, the capital

[12] Due to the Cobb–Douglas-technology, the derived results are equivalent to assuming $G_\iota = \chi_\iota Y_\iota$, where $\chi_\iota = \Theta_\iota K_\iota / Y_\iota$ and the capital coefficient K_ι / Y_ι remains constant in equilibrium. It is straightforward that an income tax rate set equal to χ would yield complete income tax financing of the public input. Nevertheless, the formulation in Eq. (25) keeps the formal analysis much simpler.

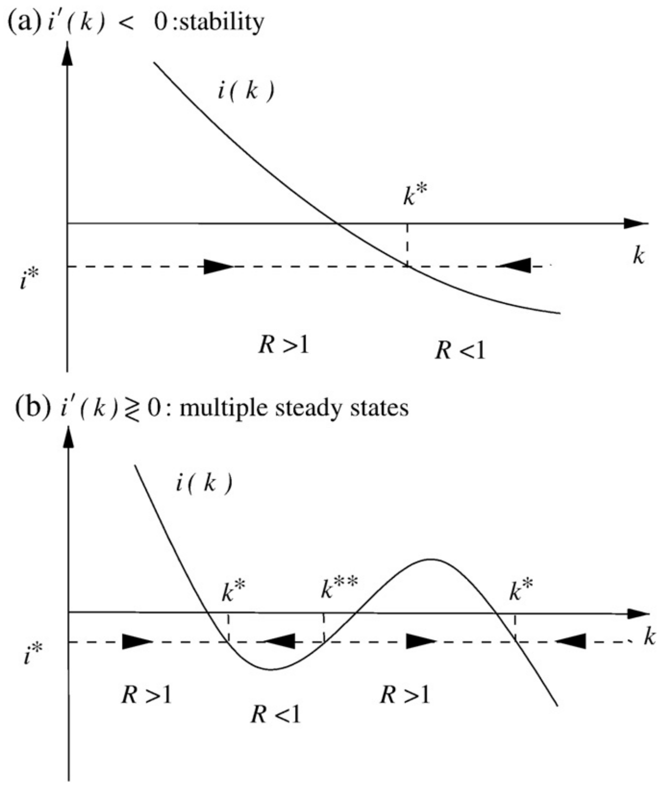

Fig. 1. Stability and multiple steady states.

distribution, k, remains constant.[13] The initial endowment with immobile labor, $-\lambda \ln l$, reflects a threshold value that does not only affect the equilibrium capital ratio, but may also have a major impact on the number of the finally resulting equilibria. The threshold value is independent of the capital ratio, k, and decreases in l and λ. In case of symmetric distribution of immobile labor, $l=1$, the term vanishes and $R \gtrless 1$ if $i(k) \gtrless 0$ The intuition for this is that, all things being equal, an increase in l increases the relative productivity in region 1. Hence, the relative capital productivity stemming from the other inputs included in $i(k)$ has to be lower in equilibrium in order to balance capital productivity in both regions ($R=1$).

Depending on the characteristics of $i(k)$, it is possible to attain either one unique equilibrium or multiple steady states, with the latter showing different stability characteristics.[14] Stable equilibria arise whenever capital ratios outside the equilibrium strive towards the equilibrium. If, in contrast, the capital ratio continuously departs from the equilibrium, the underlying equilibrium is unstable.

Within Fig. 1(a) and (b) the equilibrium capital ratios are denoted by k^* and k^{**}, respectively, and the stability implications are indicated by the arrows at the horizontal axis. The threshold value is denoted by i^*. The intuition for multiple steady states will be discussed below.

Formally, the underlying equilibrium is unstable whenever function $i(k)$ is positively sloped in the equilibrium capital (see k^{**} in Fig. 1(b)). If then, starting from the steady state capital ratio, the relative capital productivity in region 1 increases ($R>1$), the resulting capital productivity advantage in region 1 attracts investment and induces further increases of k. Hence the capital distribution departs continuously from the initial steady state and the system diverges from the unstable equilibrium. The argumentation holds analogously if, starting from an initially unstable equilibrium, k^{**}, the capital ratio is reduced and then declines continuously. If, on the contrary, the function $i(k)$ is negatively sloped for equilibrium capital ratios (see k^* in Fig. 1(a) and (b)), an increase in k reduces the ratio of capital productivities ($R<1$), thus giving rise to a productivity advantage in region 2. Then k declines and converges again to its original steady state value.

It is possible to show that within our framework either one stable equilibrium or multiple steady states result — the latter exhibiting stability characteristics as indicated within Fig. 1(b) and argued previously.[15] A more unequal distribution of immobile labor induces a shift of the threshold value away from the k-axis. Hence, if the regions sufficiently differ with respect to their endowment of immobile factors, multiple steady states will not occur even if the run of $i(k)$ would basically allow for multiple steady states. Instead, there is one stable equilibrium and the equilibrium capital ratio reflects the distribution of immobile labor, with k^* increasing in l. The simple reason is that capital and labor are complementary production factors; hence, a large amount of immobile labor causes a productivity advantage for physical capital.

4.2. Agglomeration and concentration

To analyze the regional distribution of economic activity, we now focus on agglomerations. Agglomeration means that capital is unequally distributed across regions. Following Krugman (1991), the region which holds the higher capital stock then represents the core of the entire economy, whereas the other region is the periphery. The relative size of the larger region is measured by the capital ratio, k, which describes concentration of mobile capital. The analysis will be carried out for equally distributed immobile labor, $l=1$; hence the threshold value is given by $-\lambda ln l=0$. The argumentation focuses on those determinants that affect the run of function $i(k)$ and the underlying economic effects will be discussed. The sign of $i'(k)$ determines whether agglomeration forces ($i'(k)>0$) or dispersion forces ($i'(k)<0$) prevail.

The starting point of the considerations is $i(k)$ as given in Eq. (23), and we assume that immobile labor is equally distributed, $l=1$. The derivative of $i(k)$, which decides on the domination of agglomeration or dispersion forces, results in

$$\begin{aligned}\frac{di(k)}{dk} &= \underbrace{\frac{\partial i(k)}{\partial k}}_{\text{direct effect}} + \underbrace{\frac{\partial i(k)}{\partial \tilde{g}_s}\frac{\partial \tilde{g}_s}{\partial k}}_{\text{indirect effect}} \\ &= (\alpha-1)\frac{1}{k} + (\gamma-1)(1+\varepsilon_A)\frac{\tilde{g}_s}{k}\frac{(1-\beta)(1+\beta)}{(\tilde{g}_s+\beta)(1+\beta\tilde{g}_s)} + \\ &\quad + (1+\varepsilon_A)\frac{\tilde{g}_s}{k}\frac{(\alpha(1-\beta)+\gamma\varepsilon_R)(\alpha(1+\beta)+\gamma\varepsilon_R)}{[\alpha(\tilde{g}_s+\beta)+\gamma\varepsilon_R\tilde{g}_s][\alpha(1+\beta\tilde{g}_s)+\gamma\varepsilon_R]}\end{aligned} \quad (24)$$

>From Fig. 1, we know that $i(k)$ is negatively sloped in the limits $k=0$ and $k\to\infty$. Moreover, the function $i(k)$ has an unambiguous root at $k=1$.[16] Therefore, the incidence of multiple steady states depends on the derivative of $i(k)$ in the root at $k=1$. If the slope is negative, dispersion forces dominate around the equilibrium. Consequently, the steady state with evenly distributed economic activity $k=1$ is stable. However, if the slope of $i(k=1)$ is positive, agglomeration forces dominate near to $k=1$. The equilibrium is unstable and multiple steady states arise. They are characterized by $k<1$ and $k>1$ and imply agglomeration.

It is straightforward to show that the derivative of function $i(k)$ is unambiguously negative for sufficiently low extents of intra-regional spillovers, $\varepsilon_A \to -1$. This case describes the setting where scale effects

[13] Note that identical growth rates do not imply identical absolute levels of capital or output. In general, regions will be endowed differently, and hence experience growth paths with identical growth rates but different absolute levels of output.

[14] These features about the run of the curve $i(k)$ are derived in Mathematical appendix A.

[15] See Mathematical appendix A for a proof.

[16] See the Mathematical appendix A for a proof.

are absent. There is no centripetal force, hence dispersion strictly dominates for any capital distribution, k, as given by

$$\left.\frac{di(k)}{dk}\right|_{\varepsilon_A \to -1} = (\alpha - 1)\frac{1}{k} < 0 \tag{25}$$

With an increase in intra-regional spillovers, ε_A, concentration forces arise due to productivity advantages and scale effects.[17] A rise in ε_A implies an increase in the individually available amount of the public input; hence, we have a positive effect of the aggregate capital stock on private capital returns. Moreover, scale effects come into play as the absolute size of aggregate capital affects the individually available amount of public input. A region with a relatively high aggregate capital stock, $\overline{K}_\iota$, offers a higher amount of the public input, $G_\iota = \Theta_\iota \overline{K}_\iota$. This results in more individually available public input and therefore in enhanced productivity. Comparing two regions that differ in their capital endowment, this fosters the concentration forces.

For increasing intra-regional spillovers, ε_A, the agglomeration forces may dominate in the neighborhood of $k=1$, as will be shown in the following. Provided that symmetry is given ($\theta = n = 1$), the slope of $i(k)$ in $k=1$ is given by

$$\left.\frac{di(k)}{dk}\right|_{k=1} = \underbrace{\alpha - 1}_{\text{direct effect}} + \underbrace{(1+\varepsilon_A)\frac{2\beta\gamma\varepsilon_R}{(1+\beta)(\alpha(1+\beta)+\gamma\varepsilon_R)} + (1+\varepsilon_A)\gamma\frac{1-\beta}{1+\beta}}_{\text{indirect effect}} \tag{26}$$

The first term of Eq. (26) displays the direct effect of an increase in the capital ratio on the relative capital productivity. The marginal productivity of capital is decreasing as long as the productivity impact of capital within D_ι is neglected. Since $\alpha < 1$, a rise in capital endowment goes along with a decreasing marginal product of capital. If, analogously we focus on the ratio of capital stocks, an unequal distribution of physical capital (large k) ceteris paribus leads to lower capital return in the core, $R<1$. Hence, investment is more attractive in the periphery, and this results in a decrease of k. The direct effect contributes to the convergence of the system to equally distributed physical capital, $k=1$, and tends to cut off nascent concentration.

In addition to this direct effect, there is an indirect effect of an increase in relative capital, k, on $i(k)$, which is described by the second and the third term in Eq. (26). They capture the impact of governmental activity, as incorporated within $\tilde{g}_s$, and also consider the impact of integration, β. Starting from an initial equilibrium capital ratio, any increase in k will raise the relative supply of the public inputs, $\tilde{g}_s$ (see Eq. (20)). This leads to positive effects due to the complementarity of physical capital and the public input in the production function, $Y_{\iota K_\iota G_{S\iota}} > 0$. As a consequence, the relative productivity of physical capital continues to rise, thus inducing further increases and fostering concentration. To sum up the implications of Eq. (26), the indirect effect fosters and the direct effect relaxes the concentration of economic activity.

Agglomerations arise if agglomeration forces dominate around $k=1$, and if neither agglomeration forces nor dispersion forces unequivocally prevail for all capital ratios. In general, multiple steady states arise if

$$0 < 1-\alpha < (1+\varepsilon_A)\frac{2\beta\gamma\varepsilon_R}{(1+\beta)(\alpha(1+\beta)+\gamma\varepsilon_R)} + (1+\varepsilon_A)\gamma\frac{1-\beta}{1+\beta} \tag{27}$$

and, consequently,

$$\left.\frac{di(k)}{dk}\right|_{k=1} \gtrless 0 \quad \Leftrightarrow \quad \varepsilon_A \gtrless \overline{\varepsilon}_A(\beta, \varepsilon_R) \tag{28}$$

$$\text{where} \quad \overline{\varepsilon}_A = \frac{\alpha(1+\beta-\varepsilon_R)}{\varepsilon_R - \alpha(1+\beta)} \tag{29}$$

and $\overline{\varepsilon}_A$ denotes the bifurcation point.[18] This threshold value separates the cases in which one unique and stable equilibrium (provided that $\varepsilon_A < \overline{\varepsilon}_A$) or multiple steady states (in case of $\varepsilon_A > \overline{\varepsilon}_A$) arise. Its level is crucially affected by the (exogenously given) parameters ε_R and β; both are determined by governmental decisions. Beginning with a sufficiently low level ε_A, dispersion forces dominate for all capital ratios, k, and $i(k)$ is shaped as illustrated within Fig. 1(a). If ε_A now increases until it exceeds the value of the bifurcation point as given by Eq. (29), the dynamic behavior switches toward a scenario with agglomeration. The intuition for this is that increasing intra-regional spillovers ($\varepsilon_A \uparrow$) increase local returns, thus strengthening agglomeration forces.[19] Then, the agglomeration forces dominate around $k=1$, and finally the derivative $di(k)/dk$ becomes positive; multiple steady states arise. Nevertheless, if capital is distributed more unequally across regions, the dispersion forces eventually dominate and ensure that two stable equilibria exist. Hence, agglomeration arises if (and only if) the derivative of $i(k)$, evaluated at $k=1$, is positive.

Fig. 2 provides a graphical illustration of two bifurcation diagrams. For sufficiently small ε_A the dispersion forces generally dominate and a unique equilibrium ratio k^* results. As soon as ε_A exceeds the threshold value $\overline{\varepsilon}_A$ in Eq. (29), the dynamics crucially change and multiple steady states arise.

The level of $\overline{\varepsilon}_A$ within Eq. (29) depends predominantly on the integration parameter and on the degree of relative congestion. An increase in territorial cooperation (increase in β) leads to an increase in the critical level of capital spillovers, as can be seen from

$$\frac{\partial \overline{\varepsilon}_A}{\partial \beta} = \frac{(1-\alpha)\alpha\varepsilon_R}{(\varepsilon_R - \alpha(1+\beta))^2} > 0 \tag{30}$$

Intra-regional spillovers, ε_A, have to be stronger to induce agglomeration if there is more integration. Due to the increased cooperation between the regions, the periphery can benefit from the spillovers arising in the core. Hence the agglomeration forces are weakened. If there is a close relationship between the regions (high β), the relative impact of the own region's governmental policy is weaker, and the amount of governmental input provided by the other region also affects the firm's decisions. On the other hand, in more isolated regions (low β), the own region's public input gains relatively more importance for the firm's behavior.

Even less obvious, if the public input is characterized by a higher degree of relative congestion, agglomeration is more likely to occur

$$\frac{\partial \overline{\varepsilon}_A}{\partial \varepsilon_R} = -\frac{(1-\alpha)\alpha(1+\beta)}{(\varepsilon_R - \alpha(1+\beta))^2} < 0 \tag{31}$$

The reason is that physical capital return is overestimated due to the congestion externality. Individuals do not take their impact on aggregate capital into account. When they decide about capital accumulation, they take aggregate capital as given and independent from their own decision. Therefore, individuals notice that the amount of the public input is higher in the core, but they do not take into account that capital accumulation will reduce their access to the public input. Hence, congestion leads to suboptimally high equilibrium capital accumulation and reinforces agglomeration. Consequently, the level of intra-regional spillovers, which is necessary to induce agglomeration, decreases.

[17] The slope $i'(1)$ increases in ε_A as derived in Mathematical appendix A.

[18] To derive Eq. (29), note that $\alpha + \gamma(1+\varepsilon_A) = 1$, hence $\gamma = (1-a)/(1+\varepsilon_A)$; Eq. (27) is then solved for εA. Note that to ensure the knife-edge condition of endogenous growth, the productivity of government expenditures, γ, has to be reduced whenever an increase in spillovers, ε_A, is considered. In order to prevent the preponderance of this to some extent artificial argument, we restrict to parameter settings which result in a positive denominator of $\overline{\varepsilon}_A$. In contrast to the presentation within Eq. (29), one could basically also denote the bifurcation point as $\overline{\beta}(\varepsilon_A, \varepsilon)$ or $\overline{\varepsilon}_R(\beta, \varepsilon_A)$. Qualitatively, the results would not change.

[19] This argument will be discussed in the context of Fig. 3.

Nevertheless, integration and congestion do not only influence the bifurcation point, $\bar{\varepsilon}_A$, but additionally impact the resulting concentration. Increases in ε_A may lead either to a higher or lower concentration within the equilibrium agglomerations, depending on the degree of relative congestion, ε_R, and on integration, β. Numerical simulations within the next section will help to enlighten these complex interdependencies.

5. Numerical simulations

As argued before, agglomeration only occurs if regional spillovers are sufficiently high, or, to argue more precisely, if $\varepsilon_A > \bar{\varepsilon}_A(\beta, \varepsilon_R)$ as represented by the bifurcation point within Eq. (29). Nevertheless, higher values of ε_A do not automatically result in greater concentration. The following calculations and simulations illustrate the sensitivity of the model with respect to those parameters that represent the externalities, ε_A and ε_R, as well as integration, β. We show their impact on the number of steady states as well as on concentration within Figs. 2 and 3. As far as possible, we assume symmetry, $\theta = n = l = 1$. Hence the threshold value $i^*(k) = 0$ is represented by the horizontal axis. We consider constant returns to scale in the private inputs $(\alpha + \lambda = 1)$ and make sure that the condition of endogenous growth is fulfilled $(\alpha + \gamma(1 + \varepsilon_A) = 1)$. Under these conditions (at least) one equilibrium with equal distribution of capital, i.e., $k^* = 1$, results and no agglomeration takes place within it. If, instead, multiple steady states arise, the region displaying the higher capital stock represents the core, whereas the other region may be interpreted as being the periphery. The equilibria are symmetric in the sense that one could easily change the region's indices and would have the same implications as before, but now from the point of view of the other region. Higher equilibrium values of k^* are interpreted as reflecting more concentration.

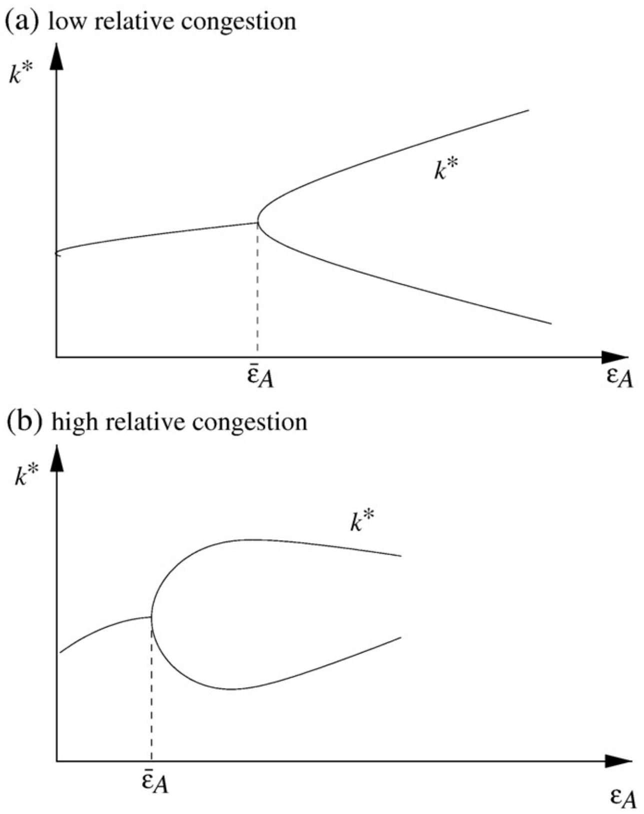

Fig. 2. Bifurcation diagram.

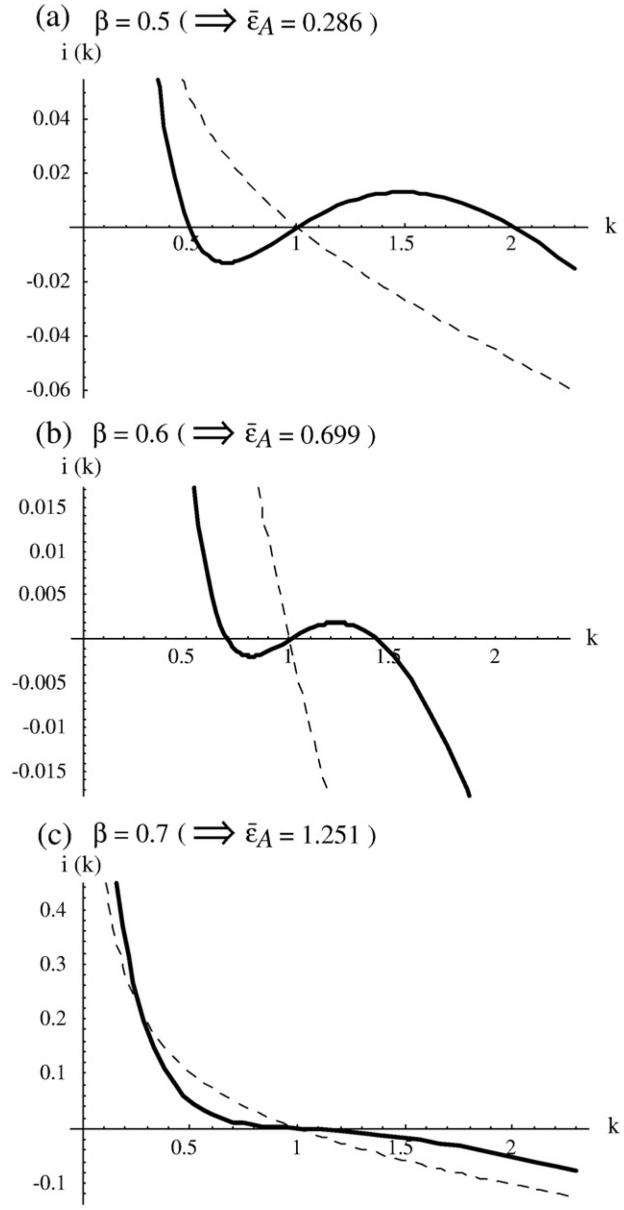

Fig. 3. The impact of integration if $\varepsilon_R = 0.5$ parameters: $\alpha = 0.3$ $\lambda = 0.7$, $\theta = 1$, $n = 1$, $l = 1 \Rightarrow i$: horizontal axis solid line: $\varepsilon_A = 0.9$, dashed line: $\varepsilon_A = -0.2$.

Fig. 3(a)–(c) plots the equilibrium capital distributions for alternative degrees of integration and assumes intermediate relative congestion, $\varepsilon_R = 0.5$. The levels of the bifurcation points, $\bar{\varepsilon}_A$, are indicated next to the respective degrees of integration. Solid lines represent high regional spillovers ($\varepsilon_A = 0.9$), while the dashed lines correspond to low levels ($\varepsilon_A = -0.2$).[20] In case of $\varepsilon_A = -0.2 < \bar{\varepsilon}_A$, the prevailing agglomeration forces are too low, capital is equally distributed across the regions, and $k^* = 1$. If, instead, $\varepsilon_A = 0.9$, agglomeration is basically possible (see Fig. 3(a) and (b)). But more integration reduces concentration (lower k^*) since then the smaller region may also benefit from the spillovers of the larger region. Consequently, capital accumulation does not move to the core. Fig. 3(c) displays a situation in which dispersion forces dominate in either case and $k^* = 1$. As argued previously, increasing integration reduces the agglomeration forces.

Fig. 4(a)–(c) emphasizes the model's sensitivity and focus on alternative levels of relative congestion for $\beta = 0.25$. Again the levels

[20] Since the simulations assume $\alpha = 0.3$, we choose this lower benchmark for ε_A to fulfill the condition $-\alpha < \varepsilon_A$.

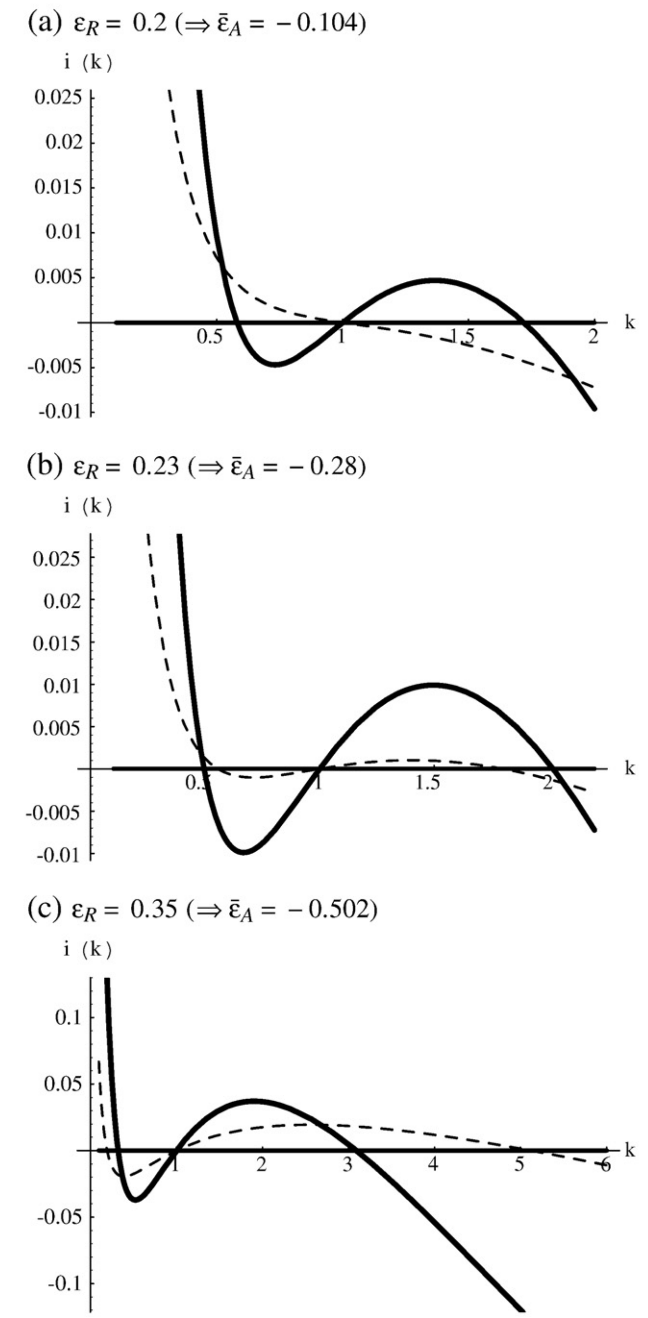

Fig. 4. The impact of relative congestion if $\beta = 0.25$ parameters: $\alpha = 0.3$, $\lambda = 0.7$, $\theta = 1$, $n = 1$, $l = 1 \Rightarrow i$: horizontal axis solid line: $\varepsilon_A = 0.9$, dashed line: $\varepsilon_A = -0.2$.

of the bifurcation points are included in parentheses below each figure. Solid and dashed lines reflect ε_A in analogy to Fig. 3, and equal distribution only arises if $\varepsilon_A < \bar{\varepsilon}_A$. The dashed function in Fig. 4(a) is one example. All other combinations of β and ε_R lead to agglomeration, and the following structure may be observed. Increasing relative congestion fosters agglomeration in either case. But note that concentration is even more pronounced for low levels of ε_A. With this, the simulations also confirm the run of the bifurcation diagram within Fig. 2(b). The intuition for this result is as follows: On the one hand, we have intra-regional spillovers that foster concentration due to ε_A. But, on the other hand, there are decreasing returns not only in private capital but also in the governmental input as discussed in the context of Eq. (26). With an increase in spillovers, ε_A, the ratio of individually available governmental inputs, $\tilde{g}_s$, increases; hence decreasing returns gain importance and reduce concentration. However, as the simulations illustrate, the total effect always implies agglomeration, not only for low but also for high values of relative congestion. Since there is a negative capital externality, which goes along with congestion, individuals overestimate private capital return. Hence, agglomeration may even become more concentrated due to an increase in congestion. Nevertheless, concentration is suboptimal, as will be shown in the following section.

6. Efficiency

In order to judge the different agglomeration scenarios, it is necessary to compare them with the social optimum. Which is the optimal degree of concentration? And is equilibrium concentration suboptimally high or low?

The efficient solution internalizes capital externalities and optimizes government expenditure rates. On the one hand, individuals neglect their influence on aggregate capital. Hence they overestimate the individually available amount of the congested governmental input. There is a negative externality of capital accumulation. On the other hand, regional governments usually neglect the productivity impact of governmental activity on the other region. There is a positive externality of governmental activity. We start with the consideration of the congestion externality. In order to evaluate the socially optimal degree of concentration, we have to take into account that private investment increases aggregate capital and hence reduces the individually available amount of the public input. If firms enlarge their truck fleet (private investment), the motorways become more crowded, and there is less infrastructure applicable for each firm. Since all firms in region ι are identical, aggregate capital is given by $\bar{K}_\iota = N_\iota K_\iota$; hence the available amount of governmental expenditures (3) amounts to

$$G_{s\iota} = \boldsymbol{\Theta}_\iota N_\iota^{1+\varepsilon_A-\varepsilon_R} K_\iota^{1+\varepsilon_A} \tag{32}$$

The social optimum is defined by the maximum individual lifetime utility. Concentration, as given by the capital distribution, k, affects the productivity and hence the income of the representative individual, $Y = Y_1 + Y_2$. Income is the base of the accumulation decision. Hence, a capital concentration k, which leads to higher income, allows faster steady state growth and thereby higher lifetime utility of the representative individual.[21] The representative individual's capital stock is given by $K = K_1 + K_2$; hence physical capital in region 1 amounts to $K_1 = kK_2$, and capital in region 2 is given by $K - kK_2$

$$\begin{aligned}\frac{\partial F}{\partial k} &= (F_{K_1} - F_{K_2})K_2 \\ &= \frac{Y_1}{k(g_s+\beta)}\left(\boldsymbol{\alpha}(g_s+\beta) + \boldsymbol{\gamma}(1+\boldsymbol{\varepsilon}_A)(g_s - \beta k)\right) \\ &\quad - \frac{Y_2}{1+\beta g_s}\left(\boldsymbol{\alpha}(1+\beta g_s)\boldsymbol{\gamma}(1+\boldsymbol{\varepsilon}_A)\left(1 - \frac{\beta g_s}{k}\right)\right)\end{aligned} \tag{33}$$

This leads to socially optimal capital accumulation determined by

$$\frac{Y_1}{Y_2 k}\,\frac{1+\beta g_s}{g_s+\beta}\,\frac{\boldsymbol{\alpha}(g_s+\beta) + \boldsymbol{\gamma}(1+\boldsymbol{\varepsilon}_A)(1-\beta k)}{\boldsymbol{\alpha}(1+\beta g_s) + \boldsymbol{\gamma}(1+\boldsymbol{\varepsilon}_A)\left(1-\frac{\beta g_s}{k}\right)} - 1 \gtrless 0 \tag{34}$$

$$\Leftrightarrow \quad i(k) + \boldsymbol{\Delta}(k) \gtrless -\boldsymbol{\lambda} \ln l \tag{35}$$

with $i(k)$, as given in Eq. (29), and $\Delta(k)$ defined as

$$\boldsymbol{\Delta}(k) = \ln\left(\frac{\boldsymbol{\alpha}(1+\beta g_s) + \boldsymbol{\gamma}\boldsymbol{\varepsilon}_R}{\boldsymbol{\alpha}(g_s+\beta) + \boldsymbol{\gamma}\boldsymbol{\varepsilon}_R g_s}\right) + \ln\left(\frac{\boldsymbol{\alpha}(g_s+\beta) + \boldsymbol{\gamma}(1+\boldsymbol{\varepsilon}_A)(1-\beta k)}{\boldsymbol{\alpha}(1+\beta g_s) + \boldsymbol{\gamma}(1+\boldsymbol{\varepsilon}_A)\left(1-\frac{\beta g_s}{k}\right)}\right) \tag{36}$$

[21] This argument is deduced more carefully in Mathematical appendix B.

$\Delta(k)$ reflects the capital externality and adjusts the ratio of private capital returns to the socially relevant relation. Δ decreases in k and goes through zero for symmetric capital distribution.[22] Furthermore, Δ is bounded from above with $\overline{\Delta} = \ln(\alpha + \gamma\varepsilon_R)$ and from below with $-\overline{\Delta}$. Therefore, the dynamics of optimal concentration are delivered according to Fig. 5.

The fact that private investment increases aggregate capital and therefore reduces the availability of the public input alters the ratio between the capital returns in the two regions. Fig. 5(b) shows that agglomeration is socially optimal. Nevertheless, concentration is sub-optimally high. Since individuals overestimate private capital returns, they react too sensitively with respect to a regional difference in capital returns. As a consequence, the degree of concentration is suboptimally high in market equilibrium.

The remaining point refers to optimal government expenditures. Public inputs such as harbors are supra-regionally productive. If region 1 increases the provision of public inputs, the productivity in both regions increases. Within the optimal choice of governmental expenditures, G_t, this impact has to be properly considered. The optimal ratio θ of regional public inputs is found by maximizing the representative individual's income, $Y = Y_1 + Y_2$, with respect to θ and taking into account that $G_1 = \theta knG_2$ and $G_2 = G - \theta knG_2$ apply. The resulting condition for optimal governmental activity is

$$\frac{\partial F}{\partial \theta} = F_{G_1}\frac{\partial G_1}{\partial \theta} + F_{G_2}\frac{\partial G_2}{\partial \theta} \overset{!}{=} 0 \quad \Leftrightarrow \quad \frac{Y_1}{Y_2}\frac{1+\beta g_s(\theta)}{g_s(\theta)+\beta} = \frac{1-\beta k^{\varepsilon_A} n^{\varepsilon_A-\varepsilon_R}}{k^{\varepsilon_A} n^{\varepsilon_A-\varepsilon_R}-\beta} \tag{37}$$

Using Eq. (18) to replace Y_1/Y_2 yields

$$\left(\frac{g_s(\theta^*)+\beta}{1+\beta g_s(\theta^*)}\right)^{\gamma} = \frac{1-\beta k^{\varepsilon_A} n^{\varepsilon_A-\varepsilon_R}}{k^{\varepsilon_A} n^{\varepsilon_A-\varepsilon_R}-\beta}(l^{\lambda}k^{\alpha})^{-1} \tag{38}$$

Within the equilibrium analysis given in the last section, the ratio of governmental activity, θ, was assumed to be arbitrarily set. Nevertheless, a regional government would decide about the amount of governmental activity, G_t, by equating marginal costs and benefits. As the homogenous good may be transformed 1:1 into governmental expenditures, marginal costs of an increase in G_t are 1. Marginal benefits result from increased productivity. It is self-evident to assume that regional governments are only concerned about the productivity in their own region. They disregard the inter-regional impact of public inputs. Usually, a regional government will only provide a harbor if the productivity gain in its own region is sufficiently high to warrant the harbor. The regional government will not take into account that, due to the harbor, other regions will experience increased productivity.

Hence, both regions equate the marginal benefits and marginal costs of governmental activity according to

$$Y_{1G_1} \overset{!}{=} 1 \quad \text{and} \quad Y_{2G_2} \overset{!}{=} 1 \quad \Rightarrow \quad Y_{1G_1} \overset{!}{=} Y_{2G_2} \Leftrightarrow \quad \frac{Y_1}{Y_2}\frac{1+\beta g_s(\theta)}{g_s(\theta)+\beta} = \frac{1}{k^{\varepsilon_A} n^{\varepsilon_A-\varepsilon_R}} \tag{39}$$

Replacing again Y_1/Y_2 with Eq. (18) leads to

$$\left(\frac{g_s(\tilde{\theta})+\beta}{1+\beta g_s(\tilde{\theta})}\right)^{\gamma} = \frac{1}{k^{\alpha+\varepsilon_A} l^{\lambda} n^{\varepsilon_A-\varepsilon_R}} \tag{40}$$

Comparing the optimal ratio of governmental activity, θ^*, and the corresponding equilibrium value, $\tilde{\theta}$, in the symmetric case yields $\theta^* = \tilde{\theta}$. The relative impact of the positive diffusion externality is of the same magnitude in each region. Hence, the ratio between governmental expenditures is unaffected. Nevertheless, the level of governmental expenditures is suboptimally low.[23] Applying this result to Fig. 5 demonstrates that selfish governmental behavior has no impact on the degree of agglomeration compared to optimal governmental activity. Nevertheless, other assumptions about regional governmental behavior could be analyzed, but this will be done in another article, since issues of political economy are not our main concern here.

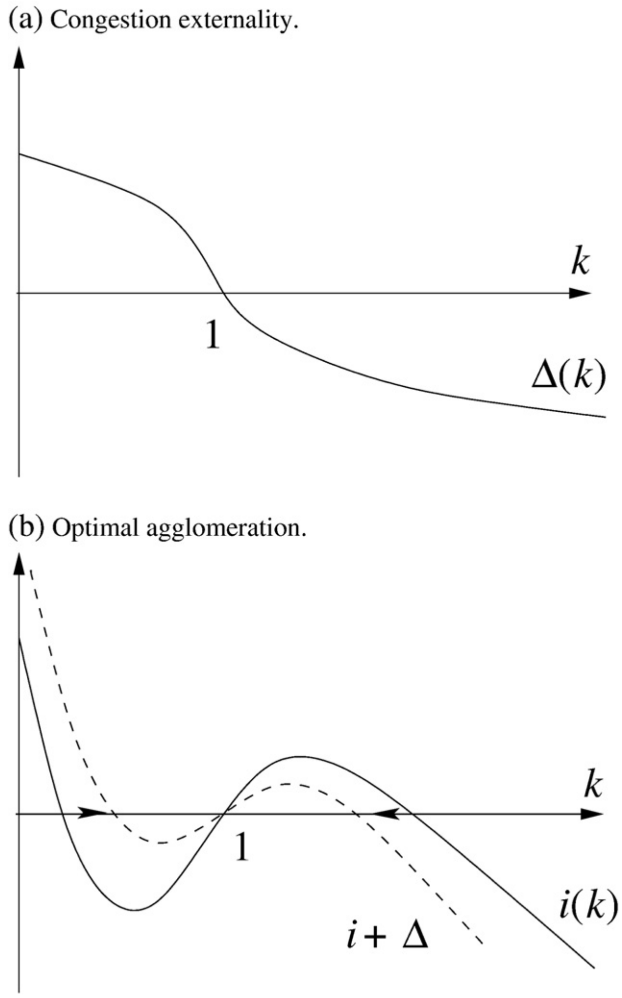

Fig. 5. Equilibrium versus optimal dynamics.

7. Conclusions

The basic objective of this paper was to analyze the impact of regional policy on the spatial distribution of economic activity. We ask, whether integration will increase concentration as usually shown in new economic geography models that interpret integration as a reduction in transport costs. And we asked whether the European regional policy to foster territorial cooperation will reach the goal to support convergence. Within the context of the model presented, regional policy includes the extent of inter-regional cooperation, as well as the type of the governmental input provided. This input affects output not only directly but also indirectly as it enhances the productivity of the other inputs. Since the governmental input is characterized by scale effects and by relative congestion, the model may be adopted to a variety of interpretations; two examples are physical infrastructure or research networks. It is shown that either

[22] The calculus is relegated to Mathematical appendix B.

[23] This is easily seen since the direct marginal returns, Y_{tG_t}, are lower than the social returns, F_{G_t}.

one unique or multiple steady states arise, with the latter showing different stability characteristics. Whether or not this leads to convergence in the sense of the European Union's regional policy goals depends upon a variety of economic conditions.

The model is very sensitive to the assumed parameter constellations, but nevertheless some basic results are derived. Integration unequivocally reduces concentration since it allows the smaller regions access to the other regions' public input and hence to benefit from its productivity impact. This result stands in strong contrast to those analyses that model infrastructure as facilitating trade. Relative congestion is associated with a negative capital externality. Capital return is overestimated and therefore aggravates concentration. As a consequence, the resulting market equilibrium ends up with suboptimally high concentration. This argument reflects the typical discussion within the growth literature about the impact of relative congestion. The effect of intra-regional capital spillovers is more complex. Agglomeration only arises if spillovers are strong enough to outweigh decreasing returns to private capital. Nevertheless, if a high level of capital spillovers applies in a situation of high relative congestion, the impact may be reversed and decrease the resulting concentration.

The model's policy implications could then be summarized as follows: More integration reduces regional disparities, while relative congestion operates in the opposite direction. These congestion externalities could be internalized by a fiscal policy that corrects for the distortions. With this, it is clear that much work is still left to be done.

Acknowledgments

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Mathematical appendix

A. Shape of $i(k)$

This first part of the appendix is concerned with the derivation of the shape of $i(k)$. The thread is as follows: The limit of i for $k=0$ is shown to be infinity, with an unambiguously negative slope. The limit of i for $k\rightarrow\infty$ is $-\infty$, and the slope eventually approaches zero. Hence, i displays at least one root. One root is shown to be at $k=1$. Hence, if the slope of i is positive for $k=1$, we have (at least) two agglomerations,[24] one for $k<1$ and one for $k>1$ (see Fig. 1).

If k tends to zero, $g_s=\theta k^{1+\varepsilon_A} n^{1+\varepsilon_A-\varepsilon_R}$ tends to zero, too. Hence, the limit of i for $k=0$ is given by

$$\lim_{k\rightarrow 0} i(k) = \underbrace{(\alpha-1)\lim_{k\rightarrow 0} ln(k)}_{\rightarrow\infty} + \underbrace{(\gamma-1)ln(\beta)}_{>0} + \underbrace{ln\left(\frac{\alpha\beta}{\alpha+\gamma\varepsilon_R}\right)}_{<0,\ \text{bounded}} = \infty \quad (41)$$

The slope of i at $k=0$ can be denoted as

$$\lim_{k\rightarrow 0} i'(k) = \lim_{k\rightarrow 0}\frac{1}{k}\Big(\underbrace{\alpha-1}_{<0} + \underbrace{(\gamma-1)(1+\varepsilon_A)g_s\frac{(1-\beta)(1+\beta)}{\beta}}_{<0,\ \rightarrow 0} + \underbrace{(1+\varepsilon_A)g_s\frac{(\alpha(1-\beta)+\gamma\varepsilon_R)(\alpha(1+\beta)+\gamma\varepsilon_R)}{\alpha\beta(\alpha+\gamma\varepsilon_R)}}_{>0,\ \rightarrow 0}\Big) = -\infty \quad (42)$$

For k going to infinity, $g_s\rightarrow\infty$, and, therefore,

$$\lim_{k\rightarrow\infty} i(k) = \underbrace{(\alpha-1)\lim_{k\rightarrow\infty} ln(k)}_{<0,\ \rightarrow-\infty} + \underbrace{(\gamma-1)ln\left(\frac{1}{\beta}\right)}_{<0} + \underbrace{ln\left(\frac{\alpha+\gamma\varepsilon_R}{\alpha\beta}\right)}_{\gtrless 0,\ \text{bounded}} = -\infty \quad (43)$$

and

$$\lim_{k\rightarrow\infty} i'(k) = \lim_{k\rightarrow\infty}\frac{1}{k}\Bigg(\underbrace{\alpha-1}_{<0} + \underbrace{(\gamma-1)(1+\varepsilon_A)\frac{(1-\beta)(1+\beta)}{(1+\beta/g_s)(1+\beta g_s)}}_{\gtrless 0,\ \rightarrow 0} + \underbrace{(1+\varepsilon_A)\frac{(\alpha(1-\beta)+\gamma\varepsilon_R)(\alpha(1+\beta)+\gamma\varepsilon_R)}{(\alpha(1+\beta/g_s)+\gamma\varepsilon_R)(\alpha(1+\beta g_s)+\gamma\varepsilon_R)}}_{>0,\ \rightarrow 0}\Bigg) = 0 \quad (44)$$

For a symmetric society, that is $k=n=\theta=1$, and hence $g_s=1$, the function unambiguously has a root

$$i(1) = 0 \quad (45)$$

Nevertheless, the slope in this root is indeterminate

$$i'(1)\gtrless 0 \quad\Leftrightarrow\quad \alpha-1\gtrless-(1+\varepsilon_A)\frac{2\beta\gamma\varepsilon_R}{(1+\beta)(\alpha(1+\beta)+\gamma\varepsilon_R)}-\gamma(1+\varepsilon_A)\frac{1-\beta}{1+\beta} \quad (46)$$

and increases in ε_A

$$\frac{\partial i'(1)}{\partial\varepsilon_A} = (\gamma-1)\frac{1-\beta}{1+\beta}+\frac{\alpha(1-\beta)+\gamma\varepsilon_R}{\alpha(1+\beta)+\gamma\varepsilon_R} = \frac{1-\alpha}{1+\varepsilon_A}+\frac{2\beta\gamma\varepsilon_R}{(\alpha(1+\beta)+\gamma\varepsilon_R)(1-\beta)}>0 \quad (47)$$

B. The social optimum

Social welfare is defined by the intertemporal utility of the representative household. In any steady state, consumption and capital grow at the same rate, φ, and the ratio of consumption to capital, $\mu=C/K$, ist constant. This leads to intertemporal utility as given by

$$U = \int_0^\infty\frac{\sigma}{\sigma-1}C(t)^{\frac{\sigma-1}{\sigma}}e^{-\rho t}dt = \frac{\sigma}{\sigma-1}\mu^{\frac{\sigma-1}{\sigma}}\int_0^\infty K(t)^{\frac{\sigma-1}{\sigma}}e^{-\rho t}dt = \frac{\sigma}{\sigma-1}(\mu K(0))^{\frac{\sigma-1}{\sigma}}\int_0^\infty e^{(-\rho+\varphi(\sigma-1)/\sigma)t}dt = \frac{\sigma(\mu K(0))^{\frac{\sigma-1}{\sigma}}}{(\sigma-1)(-\rho+\varphi(\sigma-1)/\sigma)} \quad (48)$$

An increase in steady state growth ceteris paribus enhances welfare

$$\frac{\partial U}{\partial\varphi} = (\mu K(0))^{\frac{\sigma-1}{\sigma}} > 0 \quad (49)$$

The concentration of capital, k, determines productivity in both regions and hence influences the income of the representative individual, $Y=Y_1+Y_2$. Via this channel, capital concentration determines steady state growth

$$\frac{\dot{K}}{K} = \varphi = \frac{Y_1+Y_2}{K}-\delta-\mu-\frac{G}{K} \quad (50)$$

Hence, any socially optimal concentration of capital will maximize the income of the representative individual in order to maximize

[24] We cannot exclude the possibility of more than three roots formally. Nevertheless, the numerical simulations in Section 4 show that at most three roots will occur with empirically relevant parameter settings.

steady state growth and hence intertemporal utility as used for Eq. (33).

In the following, we will analyze the slope of the function $\Delta(k)$ as given in Eq. (36), which determines the discrepancy between equilibrium agglomeration and socially optimal agglomeration. For notational convenience, we define $\Delta(k) \equiv (\Delta_1(g_s(k)) + \Delta_2(g_s(k),k))$. Hence, the slope of Δ is given by

$$\frac{d\Delta}{dk} = \left(\left(\frac{\partial\Delta_1}{\partial g_s} + \frac{\partial\Delta_2}{\partial g_s}\right)\frac{\partial g_s}{\partial k} + \frac{\partial\Delta_2}{\partial k}\right) \quad (51)$$

with

$$\frac{\partial\Delta_1}{\partial g_s} = -\frac{(\alpha(1-\beta) + \gamma\varepsilon_R)(\alpha(1+\beta) + \gamma\varepsilon_R)}{(\alpha(g_s+\beta) + \gamma\varepsilon_R g_s)(\alpha(1+\beta g_s) + \gamma\varepsilon_R)} < 0 \quad (52)$$

$$\frac{\partial\Delta_2}{\partial g_s} = \frac{\left(1-\beta^2\left(1-\alpha\gamma(1+\varepsilon_A)\frac{(1+k)^2}{k}\right)\right)}{(\alpha(g_s+\beta) + \gamma(1+\varepsilon_A)(1-\beta k))\left(\alpha(1+\beta g_s) + \gamma(1+\varepsilon_A)\left(1-\frac{\beta g_s}{k}\right)\right)} < 0 \quad (53)$$

$$\frac{\partial g_s}{\partial k} = (1+\varepsilon_A)\frac{g_s}{k} > 0 \quad (54)$$

$$\frac{\partial\Delta_2}{\partial k} = \frac{\gamma(1+\varepsilon_A)\beta}{\alpha(g_s+\beta) + \gamma(1+\varepsilon_A)(1-\beta k)} - \frac{\gamma(1+\varepsilon_A)\beta g_s}{k^2\left(\alpha(1+\beta g_s) + \gamma(1+\varepsilon_A)\left(1-\frac{\beta g_s}{k}\right)\right)} < 0 \quad (55)$$

It follows immediately that the slope of Δ is unambiguously negative.

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